Parameter sensitivity analysis on the mode shapes and resonance frequencies in an analytical vibration model of the lumbar spine

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Spinal segments with altered stiffness causing spinal deformations and low-back pain might be identifiable by comparing stiffness values between adjacent levels, since positive or negative outliers might indicate a clinically relevant condition. The stiffness matrix of a structure can be obtained by parameter estimation techniques such as model updating, which requires the construction of an analytical model, resembling the “real” structure, based on material properties and geometrical information. For the lumbar spine, these data can be obtained from quantitative computed tomography (qCT) images. However, building subject-specific models is a time consuming procedure, while it is unknown what level of accuracy is needed to obtain a reliable estimate of the stiffness matrix. Therefore, a simple vibration model of the lumbar spine was built based on lumped mass and stiffness to examine the sensitivity of the modal parameters to changes in geometry and mass properties. Input data were obtained from qCT-images of six lumbar goat spines. Subsequently, by changing the input parameters, the normalized relative sensitivities (NRS) of the first three mode shapes and corresponding eigenfrequencies to these input parameters were determined. The NRS of the mode shapes was highest for vertebral height, followed by stiffness, except in mode 2 where mass was more important than stiffness. Also, the eigenfrequencies were most sensitive to changes in vertebral height, followed by stiffness, although mass was more influential in mode 1. Changes in inertia and the location of the centre of mass had less influence on the mode shapes and eigenfrequencies. In conclusion, the most important parameters for successful model updating of the stiffness matrix are vertebral height and mass.

INTRODUCTION

Spinal deformations, neural compromise and low-back pain are thought to be caused by alterations in intervertebral stiffness, although cut-off values between healthy
and deteriorated stiffness values are not available. To overcome the lack of patients’ baseline information, data from the healthy population might be collected to serve as comparison to degenerated spinal states; however the efficacy of this approach is questionable due to the large variations in spinal mechanical properties between individuals [104]. Comparing stiffness values between adjacent spinal levels however, could help to identify damaged levels, since positive or negative outliers might indicate a clinically relevant condition. Unfortunately, obtaining stiffness information of all spinal levels requires that all levels are measured, which up to date is not clinical practice [105]. Moreover, assessment of (single level) spinal stiffness is mainly performed during spinal surgery, since it requires distraction and relaxation of neighbouring vertebrae.

Spinal segment stiffness can in principle also be assessed by structural vibration testing which requires less force on and displacement of the vertebrae, and might therefore offer a more safe approach. Structural vibration tests are performed by applying a forcing function to the spine with an electromagnetic shaker while measuring the vibration response with accelerometers or by laser-Doppler vibrometry. Previous in vitro research on single motion segments has shown that vibration responses of spinal motion segments change when the integrity of the joint is altered by removal of facet joints, by scalpel transaction of the disc, by joining adjacent vertebrae and by disc puncture [50, 59, 60, 100]. Also “natural” degeneration due to physiological processes can be detected with vibration analysis [106]. Although structural vibration testing is a promising technique, it requires knowledge of the baseline mechanical properties to detect changes in the system, and therefore it suffers from the same lack of information on healthy patient-specific stiffness values as current per-operative measurement techniques. Fortunately, the approach of comparing stiffness values of adjacent segments can also be put to use in structural vibration testing via parameter estimation techniques such as model updating. Starting point of model updating methods is a set of modal test data and an analytical model of the corresponding structure. It is assumed that the analytically computed response characteristics (e.g. resonance frequencies or eigenfrequencies and mode shapes) do not correlate well with the test data of the “real” structure, due to incorrect parameter values in the analytical model. The objective of model updating is to derive at a satisfying correlation between model and test results which provides for a physical interpretation. This is done by adjusting the parameters of the analytical model, such as stiffness and mass properties, to fit
measured data. The parameters from the updated model may then be used to evaluate the integrity of the structure, i.e. positive or negative outliers in the stiffness matrix might indicate deteriorated spinal mechanical functioning.

Patient-specific models might result in better model updating results. Imaging techniques such as quantitative computed tomography (qCT) can be used to obtain information about the material properties (e.g. bone mineral density), the distribution of material properties and the geometry of the spine, from which the mass, inertia and the location of the centre of mass (com) can be calculated [107-109]. However, it is time consuming to build a subject-specific model, while it is unclear how sensitive the modal parameters of spinal motion segments are to potential errors in geometry and mass properties, and which properties have most influence on the modal parameters. Moreover, sensitivity analysis informs on which parameter influences which modal deflection, information that can be used to optimize the model updating procedures i.e. by inserting a weighing matrix in the object function that is used to minimize the difference between the analytical modal data and the experimental modal data during the model-updating procedure. Therefore, the purpose of this study was to examine the sensitivity of the modal parameters to changes in vertebral geometry and mass properties, therewith selecting the most important parameters for successful model updating.

MATERIALS AND METHODS

Analytical Model
Keller et al. presented a basic model of the lumbar spine for vibration analysis that was adapted for the purpose of this study [110]. Briefly, a model was created in Matlab (Mathworks Inc., Natick, MA, USA) that contained five concentrated masses, the vertebrae, which were each connected by a hinge joint (the intervertebral disc and soft tissue connections). Each joint had three rotational degrees of freedom, to model either flexion-extension, lateroflexion or torsion. To mimic experimental conditions, the lower mass (representing L5) was clamped and the top mass (representing L1) was left free (Figure 5.1). The positions and accelerations of each mass were calculated relative to the positions and accelerations of the joint below each mass.
Figure 5.1. Sagittal view of the lumbar spine model from L1 to L4. The model contains five concentrated masses which are each connected by a spring $k_i$, representing the intervertebral joints L1 to L5. The lower end (L5) is clamped and the top mass (L1) is left free. The positions and accelerations $\dot{\mathbf{x}}_{1-4}$ of each centre of mass are calculated relative to the positions and accelerations of the joint below each mass (panel B). The distance between each centre of mass and the joint below is $vL$ and the distance between joints is $L$. The location of the centre of mass is given relative to the assumed rotation point in the centre of the vertebral body by $c_\gamma$. 
Using the principle of virtual work and assuming no damping, the equations of motion for the analytical model were deduced (see Appendix I) and solved in the modal space, which resulted in the eigenvalue problem:

$$\left(-\omega^2 M + K\right) \Phi = 0$$  \hspace{1cm} (5.1)

In this equation, $M$ represents the mass matrix; $K$ the stiffness matrix; $\omega$ the angular eigenfrequencies, and $\Phi$ the corresponding mode shape vector. Note that both the eigenfrequencies and the mode shapes can be obtained experimentally in “real” spines [106]. The stiffness matrix $K$ can be expanded according to:

$$K = \begin{bmatrix}
  k_1 & -k_1 & 0 & 0 \\
  -k_1 & k_1 + k_2 & -k_2 & 0 \\
  0 & -k_2 & k_2 + k_3 & -k_3 \\
  0 & 0 & -k_3 & k_3 + k_4
\end{bmatrix}$$  \hspace{1cm} (5.2)

During model updating, the values in the stiffness matrix can be adjusted to correlate model and experimental test results. The rationale is that after a satisfying fit is achieved, positive or negative outliers in the stiffness matrix might indicate damage to the intervertebral joint.

**Mass properties**

In contrast to Keller et al. [110], in this study zero mass coupling was not assumed. This implies that the mass matrix was not only assembled from the mass of the individual vertebrae, but that it also contains information about the com and mass moment of inertia. $M_i$ in Equation 5.3 represents the 6x6 rigid body matrix of vertebra $i$, and is made consistent with $K$ (see Appendix I), $m_i$ is the mass of the vertebra, $I$ is the unit matrix, $\mathbf{c}_i$ is a vector that describes the location of the com relative to the assumed rotation point at the centre of the vertebral body, and $J_i$ is the mass moment of inertia of the vertebra.:

$$M_i = \begin{bmatrix}
  m_i I & -m_i \mathbf{\ddot{c}}_i \\
  (-m_i \mathbf{\ddot{c}}_i)^T & J_i + m_i \mathbf{\ddot{c}}_i^2
\end{bmatrix}$$  \hspace{1cm} (5.3)

Values for $M$ are difficult to obtain from literature, however, they influence the eigenfrequencies and mode shapes. Thus the better $M$ approximates the “real”
M-values of the lumbar segments that were tested, the better the model response characteristics might resemble the experimental response characteristics after the updating procedure, and the more accurate the stiffness values of the individual segments might be approximated.

Since the ultimate goal of this thesis is to make vibration-based damage identification available for clinical use, M was obtained from qCT-images. Therefore, the lumbar spines of six healthy adult Dutch milk goats were harvested; the musculature was largely removed, but the ligamentous tissue was left intact. The specimens were wrapped in plastic bags and stored at -20°C. Still frozen and wrapped, the specimens were transferred to the CT-scanner (Siemens Sensation 64) and were placed in “supine” position on top of a calcium hydroxyapatite calibration phantom (Image Analysis, Colombia, KY) (Figure 5.2). Scans were performed with a slice thickness of 0.6 mm, matrix size of 512 x 512 and settings of 500 mA and 140 kV.

Figure 5.2. The specimens were placed in “supine” position on top of a calcium hydroxyapatite calibration phantom that contained five regions with a mass density of -100 mg·cc\(^{-1}\), 0 mg·cc\(^{-1}\), 50 mg·cc\(^{-1}\), 100 mg·cc\(^{-1}\) and 200 mg·cc\(^{-1}\) that were visible on each slice.
Subsequently, the CT-images were imported into a commercially available software program (Scan IP and ScanFE, Simpleware Ltd. Exeter, UK) in which the images could be segmented and converted to a 3D-volume mesh of the vertebrae. Using a semi-automated segmentation technique the vertebral surface on each sagittal slice was described as accurately as possible, transversal and coronal images were used to check for errors. Discontinuities in the surface were filled in manually to result in continuous objects and finally the images were meshed using tetrahedral elements.

To assign material properties to the tetrahedral elements of the volume mesh, the CT-attenuation data in Hounsfield units were converted to values of bone mineral density using the calibration phantom. The calibration phantom contained five regions with a mass density of -100 mg·cc⁻¹, 0 mg·cc⁻¹, 50 mg·cc⁻¹, 100 mg·cc⁻¹ and 200 mg·cc⁻¹ that were visible on each slice (Figure 5.2). Each region was segmented in a similar way as the vertebrae, the mean Hounsfield value was determined, and the linear relation between Hounsfield units and density was calculated. Based on greyscale

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**Figure 5.3.** Measures of vertebral height (h) and the position of the centre of mass (com) in y-direction (c) relative to the assumed rotation point in the centre of the vertebral body, that were obtained from the CT-images. The position of the centre of mass was constructed from the vertebral body depth (vd) and the depth of the spinal canal (cd).
values, eight “materials” were stored for each vertebra [109], and the values for each of these material groups were corrected based on the calibration relation. After the material properties were assigned, the data was exported to MSC Patran (MSC software corporation, Santa Ana, USA) to calculate the mass, inertia and the com of each vertebra.

**Vertebral geometry**
In addition to the mass properties, also information on vertebral geometry and disc height is needed to construct the analytical model (see Appendix I). Therefore, vertebral height and disc height were measured on the CT-images. The height of the vertebral body was measured on the mid-sagittal image at the anterior side of the vertebral body and at the posterior side and averaged. Also disc height was measured anteriorly and posteriorly and averaged.

Furthermore, information on the location of the com relative to the assumed rotation point was required for the construction of the model. Although somewhat crude, the assumption was made that the rotation point was located in the geometrical centre of the vertebra, and that the com lies in the middle of the spinal canal, as illustrated in Figure 5.3. To construct the distance between these two points, for the y-direction, the maximum vertebral body depth and the maximum depth of the spinal canal were measured and divided by two. For the x-direction and for the z-direction the distance between com and rotation point, $c_x$ and $c_z$, were assumed zero.

**Sensitivity analysis**
To analyse the sensitivity of the natural frequencies and the mode shapes in flexion-extension and in lateroflexion to changes in the mass properties and vertebral geometry, the modal parameters were first obtained with a nominal model. Thereafter, the input to the model was changed one parameter at a time and the modal parameters were recalculated. The input parameters that were changed were mass $m_i$, moment of inertia for flexion-extension $J_{zz}$ and lateroflexion $J_{yy}$ and vertebral body height $h_i$, which were enlarged by their standard deviation as obtained between the spines that were scanned (Table 5.1). The sensitivity of the modal parameters for the location of the com was tested by increasing the value of $c_i$ with its standard deviation in y-direction, and for the x- and z-direction with 5 mm. For comparison, also the sensitivity of the modal parameters for variations in stiffness was calculated. This was done by increasing the
nominal stiffness values $k_i$ of 11 N·m·rad$^{-1}$ for flexion-extension and of 7 N·m·rad$^{-1}$ for lateroflexion, one by one by 5 N·m·rad$^{-1}$. These stiffness values were averages and standard deviations obtained in quasi-static 4-point bending tests of six comparable healthy goat specimens (unpublished data).

Since the model parameters have different units and vary over different ranges, their sensitivities cannot be compared directly. Therefore, the relative change ($\Delta P$) in the input en response parameters were calculated from the original values ($P_{\text{nominal}}$) and the original values summed with their standard deviation ($P_{\text{new}}$) according to Equation 5.4. Subsequently, the normative relative sensitivities ($NRS$) were assessed as the ratio between the relative change in the model parameters ($\Delta P_{\text{input}}$) and the relative change in the response parameters ($\Delta P_{\text{response}}$) according to Equation 5.5.

\[
\Delta P = \left( \frac{P_{\text{new}} - P_{\text{nominal}}}{P_{\text{nominal}}} \right) \times 100\% \tag{5.4}
\]

\[
NRS = \frac{\Delta P_{\text{response}}}{\Delta P_{\text{input}}} \tag{5.5}
\]

RESULTS

The mass properties, relevant vertebral geometry measures and disc height of L1 to L5 of the six goat spines are given in Table 5.1.

Figure 5.4 shows an example of how the first, second and third mode shape in flexion-extension change after the model input was changed. The left panels show changes to vertebral height and the right panels show changes to stiffness. These graphs show that variations in height and stiffness have a larger influence on mode 3 than on mode 1. Furthermore, it can be observed that in mode 1 only perturbations of the stiffness $k_4$, which is the spring that is closest to the fixed end, have an effect on the amplitude of the mode, while in mode 3, the largest effect on the mode shape is caused by perturbations of $k_1$.

Figure 5.5 shows the $NRS$-values for the mode shapes (left panels) and the eigenfrequencies (right panels), for flexion-extension. In these graphs, the effect of variations in the input parameters on the mode shapes and eigenfrequencies can be
Table 5.1. Means and standard deviations (SD) for the mass properties and vertebral geometry of 6 goat spines from which the vibration model was constructed and that were used for the sensitivity analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Level</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mass [kg]</strong></td>
<td>L1</td>
<td>0.051</td>
<td>0.013</td>
</tr>
<tr>
<td></td>
<td>L2</td>
<td>0.057</td>
<td>0.013</td>
</tr>
<tr>
<td></td>
<td>L3</td>
<td>0.062</td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td>L4</td>
<td>0.062</td>
<td>0.014</td>
</tr>
<tr>
<td><strong>J_{zz} \times 10^{-6} \text{ kg m}^2</strong></td>
<td>L1</td>
<td>15.404</td>
<td>6.707</td>
</tr>
<tr>
<td></td>
<td>L2</td>
<td>21.653</td>
<td>8.467</td>
</tr>
<tr>
<td></td>
<td>L3</td>
<td>26.045</td>
<td>11.746</td>
</tr>
<tr>
<td></td>
<td>L4</td>
<td>26.314</td>
<td>9.960</td>
</tr>
<tr>
<td><strong>J_{yy} \times 10^{-6} \text{ kg m}^2</strong></td>
<td>L1</td>
<td>15.574</td>
<td>7.572</td>
</tr>
<tr>
<td></td>
<td>L2</td>
<td>23.104</td>
<td>9.106</td>
</tr>
<tr>
<td></td>
<td>L3</td>
<td>27.352</td>
<td>12.384</td>
</tr>
<tr>
<td></td>
<td>L4</td>
<td>28.559</td>
<td>10.968</td>
</tr>
<tr>
<td><strong>Vertebral height [m]</strong></td>
<td>L1</td>
<td>0.035</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>L2</td>
<td>0.036</td>
<td>0.004</td>
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<tr>
<td></td>
<td>L3</td>
<td>0.038</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>L4</td>
<td>0.038</td>
<td>0.004</td>
</tr>
<tr>
<td><strong>Vertebral depth [m]</strong></td>
<td>L1</td>
<td>0.014</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>L2</td>
<td>0.014</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>L3</td>
<td>0.014</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>L4</td>
<td>0.014</td>
<td>0.002</td>
</tr>
<tr>
<td><strong>Disc height [mm]</strong></td>
<td>L1-L2</td>
<td>4.417</td>
<td>2.843</td>
</tr>
<tr>
<td></td>
<td>L2-L3</td>
<td>4.342</td>
<td>2.115</td>
</tr>
<tr>
<td></td>
<td>L3-L4</td>
<td>4.000</td>
<td>2.132</td>
</tr>
<tr>
<td></td>
<td>L4-L5</td>
<td>4.000</td>
<td>1.809</td>
</tr>
<tr>
<td><strong>Spinal canal depth [m]</strong></td>
<td>L1</td>
<td>0.010</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>L2</td>
<td>0.011</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>L3</td>
<td>0.011</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>L4</td>
<td>0.011</td>
<td>0.002</td>
</tr>
</tbody>
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compared directly. The NRS of the mode shapes is highest for vertebral height (on average 0.27, 1.30 and 0.87 for mode 1, 2 and 3 respectively), followed by stiffness (on average 0.11, 0.24 and 0.45), except for the second mode, where the effect of mass was larger than for stiffness (on average 0.02, 0.54 and 0.29). The third mode was also sensitive to changes in $c_y$, but mode 1 and 2 were not (0, 0.10 and 0.24). Inertia and $c_x$ had negligible effects on the mode shapes (all values < 0.01). For the eigenfrequencies, the NRS was also highest for vertebral height (on average 0.21, 0.19 and 0.10), followed by stiffness in mode 2 and 3 (all 0.10), and mass in mode 1 (0.11, 0.09 and 0.06). Also the eigenfrequency of mode 3 was sensitive to changes in $c_y$ (0.01, 0.03 and 0.06). The NRS of inertia and $c_x$ were zero.

The results on the sensitivities were almost similar for lateroflexion as for flexion-extension, although the absolute NRS-values differed somewhat. Results in lateroflexion were different with respect to the inertia and $c_i$. In lateroflexion $c_z$ had no influence where $c_y$ had some influence on the mode shapes in flexion–extension. $J_{yy}$ had some influence on the mode shapes (average NRS 0, 0.18 and 0.22) and eigenfrequencies (0.01, 0.04 and 0.08), while $J_{yy}$ had no influence in flexion-extension.

Finally, Figure 5.5 also shows the relative effect of the individual model parameters on the mode shapes and eigenfrequencies. For example, it shows that L4 in the first mode is more sensitive to changes in $k_4$ than to changes in $k_1$.

**DISCUSSION**

Structural vibration testing might provide a new approach toward evaluation of spinal structure and functioning when the technique can be developed for use in patients. This study showed how subject-specific information on vertebral geometry and mass properties can be derived from qCT-images and used to construct an analytical vibration model of the goat lumbar spine. Secondly, the sensitivity of the model for errors in the input parameters was tested, which showed that the mode shapes and eigenfrequencies for bending are mainly sensitive to vertebral height, stiffness and vertebral mass. This information can be used to facilitate future model-updating procedures for the detection of structural damage of intervertebral joints, and tells which individual parameters are critical to estimate the intervertebral stiffness.
Figure 5.4. The first, second and third mode shape in flexion-extension (top, middle and bottom panels respectively). The fat grey line indicates the mode shape of the nominal model, the other lines show how the mode shapes change when vertebral height (left panels) and the stiffness values (right panels) were varied.
The sensitivity of the modal parameters to variations in stiffness was used for comparison; when the $NRS$ of stiffness would have been far greater than the $NRS$ of the other parameters in the model, it would not be necessary to find accurate subject-specific data on mass properties and vertebral geometry. On the other hand, when the sensitivity for the modal parameters would have been high for all parameters but the stiffness, model updating to detect possible defects of the intervertebral joint by identifying deviating stiffness values might not be feasible. The results showed that the mode shapes and the eigenfrequencies were sensitive to perturbations in stiffness. Moreover, the sensitivity to vertebral height was high indicating that measuring these heights to parameterize the model will facilitate estimation of the stiffness.

A basic analytical vibration model was used here. More advanced dynamical models have been build of the spine [111-113], but their complexity requires a multitude of parameters that need to be measured as model input, while each parameter will contain measurement error. Alternatively, these parameters might be estimated by the model-updating procedure. However, the ratio between experimental data fed to the model, and the number of model parameters might become unfavourable since with too many parameters, the optimisation will generate physiologically meaningless results. Furthermore, a large number of parameters to be included in the process of minimising the error between experimental and analytical data is computationally expensive. Valentini describes in his study that small differences are found between the results of full body and partial models of the spine and that multi-body models are able to mimic vibrational behaviour of reasonable complexity [114].

A limitation of this study is that only one degree of freedom was allowed per motion segment (only flexion-extension or lateroflexion), while coupled motions occur in the human spine. This may have influenced the sensitivity of the modal parameters for the off-axis location of the com, i.e. when the com lies off the anteroposterior axis of symmetry and motions in all degrees of freedom would be allowed, the segment would likely also show torsion and lateroflexion. This implies that prior to model updating, the experimental mode shapes have to be analysed carefully, so that the mode shapes that represent coupled motions are not mistaken for pure flexion-extension or lateroflexion and are fitted with analytical mode shapes that represent pure bending.
Figure 5.5. Normalized relative sensitivity (NRS)-values for the mode shapes (left panels) and the eigenfrequencies (right panels) for mode 1 to 3 for flexion-extension. The input parameters of the model were changed one at the time by their standard deviation, and the effect on the deflections $u_1$-$u_4$ of corresponding vertebrae L1-4 and on the eigenfrequencies was calculated. The parameters that are shown here are $L_{1-4}$, $k_{1-4}$, $h_{1-4}$ and $c_{1-4}$, which corresponds to vertebral mass, stiffness, height and location of the centre of mass relative to the assumed rotation point in the centre of the vertebra.
The present sensitivity analysis was based on data from goat spines, as these are frequently used as animal models of the human spine. Since the geometry of the goat vertebra is different from the human vertebra, especially considering the shape and mass of the posterior elements, the results from this study have to be translated carefully to human spine research. Furthermore, it is likely that the standard deviations of parameter values are larger for human spines. Still, this study presents a new approach towards damage identification of the spine using structural vibration testing and model-updating techniques. The derivation of model input parameters from qCT and the analysis of their influence on the eigenfrequencies and mode shapes can easily be repeated for human spines.

**CONCLUSION**

The results from this study show that the most important parameters for successful model updating of the stiffness matrix are vertebral height and mass. Moreover, since the modal parameters were equally sensitive to changes in intervertebral stiffness, the model that was presented can be used for model updating of the stiffness matrix in order to identify intervertebral joints that were damaged.

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Authors would like to thank M. Erber, M. Gigengack and Prof. Dr. C. van Kuijk from the Department of Radiology of the VU University Medical Centre Amsterdam for their efforts in obtaining and analyzing the qCT-images of the goat spines.
Development of vibration-based damage identification in the lumbar spine

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Vibration testing might allow the identification of damaged or degenerated segments in the lumbar spine. Changes in the eigenfrequencies can point at changes in stiffness, and mode shapes can be used to localize the damage. Model updating makes use of modal parameters by fitting numerical vibration data to experimental vibration data by adjusting the parameters (such as stiffness) of the numerical model. The purpose of this study was to investigate whether model updating can be used to estimate the stiffness of individual motions segments and identify degenerated joints within the lumbar spine. The aim was to identify intervertebral joints with a decreased bending stiffness caused by experimentally induced disc degeneration. Six goat specimens containing 2 healthy and 2 degenerated intervertebral discs at random levels underwent quasi-static 4-point bending tests and vibration tests. Quasi-static stiffness estimates were obtained from the load-deflection curves and dynamic stiffness estimates by model updating. Also vertebral mass and the height of the top vertebra were estimated by model updating. The correlations between static and dynamic stiffness estimates in flexion-extension and in lateroflexion were low and not significant ($R=0.07$, $p=0.74$ and $R=0.09$, $p=0.72$), most probably due to inaccuracies in the estimated vertebral mass obtained by model updating. Although the degenerated levels could not be identified, the results indicate that the proposed methodology can work, provided that independent estimates of vertebral mass are obtained to parameterize the model.

INTRODUCTION

Damage and injury to spinal motion segments alter the intervertebral stiffness and might hamper the mechanical functioning of the spine. Traditionally, examination of spinal functioning is performed by physical examination. When no clear diagnosis can be made or when a differential diagnosis is needed, the examination can be extended by imaging techniques such as radiographs and magnetic resonance imaging (MRI). But even when abnormalities are present on the images, no information on mechanical
functioning is provided. Unfortunately, up to date assessment of mechanical functioning, and more specifically quantification of spinal segment stiffness is not possible in patients [105].

Vibration testing can in principle be used to estimate spinal segment stiffness. The fundamental idea behind vibration-based damage identification is that damage causes alterations in the physical properties of a structure (mass, damping and stiffness), which will lead to detectable changes in the modal properties (eigenfrequencies, damping and mode shapes) [1]. Previous research, also in the spine, has focussed on the relation between damage (loss of stiffness) and the change in the eigenfrequency [50, 100, 115]. However, for systems with more degrees of freedom, using eigenfrequencies alone is not sufficient. Eigenfrequency changes depend on the square root of the alteration in stiffness, but they contain no information on the location of such change within the spine [115]. To localize structural damage, mode shape methods have been developed. Mode shapes contain local information and therefore are sensitive to changes in the element stiffness of a structure. Model-based mode shape methods make use of a numerical model that resembles the real structure. The parameters of this model (density, rotational stiffness, geometry) are updated to tune the numerical model to the real structure. Updating of the model variables is done iteratively, by minimizing differences between analytical and experimental vibration data.

Since the spine has multiple degrees of freedom, mode shape methods, such as model updating, are needed to obtain information on alterations in segmental stiffness. Model updating, in contrast to techniques such as structural health monitoring [59] does not necessarily start with a model of the intact structure, which is an advantage, since baseline information on healthy stiffness values will not normally be available. However, the stiffness of motion segments is correlated quite highly within spines [105], and changes only gradually over spinal levels in healthy spines [78]. Therefore, injured segments might be identified from positive or negative outliers in the stiffness matrix that results from model updating.

Vibration-based damage identification in the lumbar spine, using model-updating methods without baseline information on intervertebral stiffness, has to our knowledge not been attempted previously. Since this is the first study to do so, experimental vibration data were obtained in a controlled in vitro condition from lumbar goat spines.
and the simplest possible analytical vibration model that is able to describe the modal characteristics with sufficient precision was used. Although several finite element models suitable for vibration analysis have been presented in the literature [111, 112, 116], these models suffer from their complexity. They require multiple input parameters that can all be expected to contain some error, and the high complexity of the model hampers interpretation of the outcomes. The model of a goat lumbar spine that was used here was introduced previously [117] and consists of four lumped masses that represent the vertebrae (see Appendix I). Each vertebra is connected to the next vertebra with three rotational springs that represent the intervertebral joints and model flexion-extension, lateroflexion and torsion, although the latter was not assessed in this study. The aim of this study was to investigate whether model updating of the lumbar spine can be used for vibration-based damage identification in the lumbar spine. This methodology does not strive for a detailed description of injury mechanisms, but aims to identify the intervertebral joints with a different bending stiffness compared to adjacent segments.

**MATERIALS AND METHODS**

**Specimens**
The lumbar spines of six healthy adult Dutch milk goats were injected with chondroitinase-ABC at random levels to create mild degeneration. This degeneration model was used before to successfully create degeneration, and the research protocol was approved by both a Scientific Board and the Animal Ethics Committee of the VU University Medical Centre [118]. The injection sites could include any level between T13-L1 and L5-L6. After 12 weeks, the spines were harvested and the musculature was carefully removed. MRIs were obtained of all spines to exclude any unexpected abnormalities. The spines were sectioned so that each specimen contained two healthy and two injected discs and it was ensured that the two injected discs were randomly present among the four discs of the specimens. The posterior elements were removed with a band saw so that only the mechanical properties of the intervertebral disc would be assessed. Also parts of the transverse processes were removed (Figure 6.1).

**Statiscal mechanical tests**
To obtain stiffness values for comparison to the stiffness matrix from model updating and to test if the chondroitinase-ABC injections had caused mild degeneration, quasi-
Figure 6.1. The goat specimens (left image) that were used contained no posterior elements, part of the transverse processes were removed and the top vertebra was cut in half to free the specimen from its embedding. These actions were virtually performed on the finite element models (right images) to tune the finite element vertebrae to the “real” vertebrae and to calculate the mass properties.
static 4-point bending tests were performed on the specimens. The mechanical testing procedure and equipment was similar as previously reported [106], except that three load cycles in flexion-extension and lateroflexion were applied from -1 N·m to +1 N·m at a rate of $0.5^\circ\cdot s^{-1}$. The data of the third cycle were used for analysis and the static stiffness of the four discs was obtained from the load-deformation curves according to the method of Smit et al. [36].

**Vibration tests**

After the mechanical tests, the top cup in which the specimen was embedded for the static 4-point bending tests was carefully removed from the specimen by sawing through the top vertebra as close as possible to the embedding. The specimen remained embedded in the bottom cup, which was mounted to the measurement table. Vibration tests and equipment have been described previously [106]. Here, the shaker was attached to a screw in the upper vertebra that was positioned at the ventral side in line with the mediolateral symmetry axis of the vertebra. To excite flexion-extension, the shaker was attached in-line with the screw. To excite lateroflexion and torsion, the shaker was attached to the screw at an angle of $90^\circ$. During every trial, the motions of the most distal points on the left and right transverse processes of the four vertebrae were measured in anteroposterior direction and in mediolateral direction using laser-Doppler vibrometry (Polytec, OFV 303 sensor head; OFV 3001 vibrometer controller). From the resulting FRF, the eigenfrequencies and vibration modes were determined by curve-fitting techniques using commercially available modal analysis software (ME’scope VES 5.0, Vibrant Technology).

**Model parameters**

The analytical vibration model that was created in Matlab (Mathworks Inc.) calculates mode shapes and eigenfrequencies based on model input parameters (see Appendix I). These are vertebral height, disc height, second moment of inertia of the vertebrae, the location of the centre of mass of the vertebrae relative to the point of rotation of the vertebrae and the vertebral mass. The sensitivity analysis that was performed revealed that the model parameters are sensitive to changes in vertebral height and vertebral mass [117]. Therefore, accurate values for these model parameters are required to estimate the stiffness matrix by model updating.
Previously, mass properties of goat vertebrae were calculated from 3D-finite element models based on quantitative computed tomography images [117]. However, for this study only MRIs were available. Therefore, for each goat and each level, the average of anterior and posterior vertebral height and the average of anterior and posterior disc height from the MRIs were used as model input, and vertebral mass $m_i$ was calculated from the vertebral height $h_i$ according to Equation 6.1. This relation was obtained from linear regression analysis of the finite element data from the previous study (Figure 6.2) [117].

$$m_i = 5.00 \cdot h_i - 0.12$$  \hspace{1cm} (6.1)

Although the goat spines that were used in this study were similar to the spines used in the previous study, the posterior elements, part of the transverse processes and half of the top vertebrae were removed here. Therefore, the finite element models were tuned to the spines in this study by virtually removing the corresponding parts from the finite element models and recalculating the mass properties (Figure 6.1). The mass of the top vertebra was estimated to be reduced by on average 68.46% and that of the other vertebrae by 48.68%. Since the modal parameters are only moderately sensitive to changes in inertia and the location of the centre of mass relative to the assumed point...
of rotation of the vertebra [117], the average values for each level obtained from the adjusted finite element models of the six goat spines were used for model updating.

**Model updating**

In model updating, an optimisation problem is designed in which the differences between the experimental and analytical model are minimized by adjusting the uncertain model parameters. Each model updating step began by correcting differences in scale between the analytical and the experimental mode shapes, that in part originated from the fact that the experimental data were obtained from measurements in which only the translational degrees of freedom could be measured, while the analytical model has only rotational degrees of freedom (see Appendix I). Furthermore, an absolute scaling of the experimental mode shapes on the input force was not available, because these data could not be imported to the modal analysis software. Therefore, for each experimental mode and for each analytical mode the point with the largest displacement was selected and all other displacements were divided by this maximum to normalize the modes to one in their maximum displacement [37].

Thereafter, the stiffness of the intervertebral joints, the mass of the vertebrae and the height of the top vertebrae were estimated by model updating. To ensure physically meaningful updating, parameter values were allowed to be changed within a narrow trust region. For stiffness and mass, the upper and lower boundaries of the trust region were ± 2 standard deviations as calculated from previous data from similar goat specimens (unpublished data); the height was allowed to be adjusted within 25% to 75% of the intact height. Adjustment of stiffness, mass and height of the top vertebra was performed by minimizing the residuals of the eigenfrequencies $r_f$ and the mode shapes $r_s$:

$$
 r_f = \sum_{i=1}^{N} \alpha_i \left( \frac{\lambda_i^a - \lambda_i^e}{\lambda_i^e} \right)^2 
$$

(6.2)

$$
 r_s = \sum_{i=1}^{N} \beta_i - \beta_i \left( \left( \phi_i^a \right)^T \cdot \phi_i^e \right)^2 
$$

(6.3)

$$
 \min \varepsilon = r_f + r_s 
$$

(6.4)
In these equations $\lambda^a_i$ is the analytical eigenfrequency of the i-th mode, $\lambda^e_i$ is the experimental eigenfrequency of the i-th mode, $\Phi^a_i$ is the analytical mode shape of the i-th mode and $\Phi^e_i$ is the experimental mode shape of the i-th mode. The $\alpha_i$ and $\beta_i$ terms allow a degree of confidence to be expressed in the accuracy of the various modal properties. Their values were derived from the sensitivity analysis described in the previous chapter, and by choosing values that were likely to generate the most optimal fit. For most goats the values for $\alpha_{1,3}$ were 25, 10 and 1 respectively, and 30 for $\beta_{1,3}$, but the first mode shape of goat 6 appeared odd by visual inspection, probably due to measurement error, and therefore $\alpha_1$ was set at 1. $\epsilon$ is the term that was minimized.

Minimizing $\epsilon$ was achieved by simulated annealing. Simulated annealing is a generic probabilistic metaheuristic for the global optimization problem of locating a good approximation to the global optimum. At each step, the heuristic considers a neighbouring state of the current state, and probabilistically decides between moving the system to the new state or staying in the present state. In contrast to simple heuristics that move by finding best neighbour after best neighbour and stop when they have reached a solution that has no neighbours that are better solutions, metaheuristics use the neighbours of a state as a way to explore the solutions space and can accept worse solutions in their search in order to accomplish that. This means that the search will not get stuck to a local optimum and if the algorithm is run for an infinite amount of time, the global optimum will be found. Here, the search was set to continue until the difference between the current solution and the previous solution was less than $1E^{-6}$ or until the maximum number of function evaluations reached $5E^6$.

The goodness of fit was checked by calculating the Modal Assurance Criterion ($MAC$) between the mode shapes of the adjusted model and the experimental mode shapes according to Equation 6.5. The $MAC$ reaches values between 0 (no fit) and 1 (optimal fit).

$$MAC_i = \frac{\left(\Phi^a_i \cdot \Phi^e_i\right)^2}{\left(\Phi^a_i \cdot \Phi^a_i \cdot (\Phi^e_i)^\top \cdot (\Phi^e_i)^\top\right)^{1/2}}$$

(6.5)

**Check of the model updating procedure**

To check whether the model-updating procedure worked appropriately, the procedure was first performed with simulated modal data. To this end, an analytical vibration
Development of vibration-based damage identification in the lumbar spine

A model was constructed in Nastran (MSC Software) that was analogous to the Matlab vibration model. Initially, vertebral height was set at 18, 36, 38 and 38 mm from top to bottom, vertebral mass was 17, 30, 30 and 32 gram, and the stiffness values of the springs were set at 11 N·m·rad\(^{-1}\). With these values, initial modal characteristics were calculated. First, only the stiffness values were estimated by updating the Matlab model. Second, also vertebral masses were estimated. Thereafter, the Nastran model was used to generate two more data sets: the first in which the stiffness value of the second hinge was decreased to 5 N·m·rad\(^{-1}\) and of the third hinge was increased to 13 N·m·rad\(^{-1}\), and the second in which the masses of the four vertebrae were increased (to 20.4, 36, 36 and 38.4 gram respectively). Vertebral height was not varied.

Figure 6.3. The model updating results from the three simulated data sets. The simulated stiffness is represented by the black squares, the estimated stiffness by the grey squares, the simulated masses by the black circles and the estimated masses by the grey circles. The top left panel shows the results when only stiffness was estimated, the top right panel when also mass was estimated. The bottom left panel shows the fitting results of the model with altered stiffness values for hinge 2 and 3, and in the bottom right panel the masses of the model were altered.
Figure 6.4. The model updating results from the experimental data for flexion-extension. The quasi-static stiffness estimates of the four intervertebral discs are represented by the black squares and the injected levels are marked by the squares with the black face colour. The grey squares represent the dynamic stiffness estimates and the circles the estimated masses of the vertebrae. Vertebral height is indicated by the dots, #1 is the estimated vertebral height, the other heights were measured on MRI.
RESULTS

Simulated data
The model updating results from the simulated data sets are presented in Figure 6.3, where the simulated stiffness is represented by the black squares, the estimated stiffness values by the grey squares, the simulated masses by the black circles and the estimated masses by the grey circles. The top left panel shows that estimating only stiffness values generates excellent results. When masses are also estimated (top right panel) differences between simulated parameters and estimated parameters increase. More specifically, the masses and the stiffness values were overestimated by the model updating. Still, the estimated stiffness values were closely correlated to the input values. Pearson’s correlation coefficient for stiffness was $R=0.99$, $p<0.01$ and for mass $R=0.97$, $p<0.01$. The $MAC$ was 1.0 for all three modes in all test cases.

Experimental data
The static stiffness values of the four intervertebral discs are represented by the black squares in Figure 6.4 (flexion-extension) and Figure 6.5 (lateroflexion). The injected levels are marked by the squares with the black face colour. Statistical tests (generalized estimation equations, GEE) revealed that on group level, the injected discs had a significantly lower stiffness in lateroflexion than the discs that were not injected ($p<0.01$), but not in flexion-extension ($p=0.12$). Unfortunately, injected levels did not clearly deviate in stiffness from adjacent segments.

The grey squares in Figure 6.4 and 6.5 represent the stiffness values that resulted from model updating. In flexion-extension, it appears that the dynamic stiffness estimates closely approximate the quasi-static stiffness estimates in discs 2 and 3, but not in discs 1 and 4. Pearson’s correlation coefficient between the quasi-static stiffness estimates and the dynamic stiffness estimates was $R=0.07$, $p=0.74$, while the correlation between the estimates for discs 2 and 3 was $R=0.72$, $p<0.01$. In lateroflexion, the two stiffness estimates at the free end and at the clamped end appeared to converge better than in flexion-extension. However, also here the correlation between quasi-static and dynamic stiffness estimates was only $R=0.09$, $p=0.72$. The correlation between stiffness estimates for discs 2 and 3 was $R=-0.02$, $p=0.95$. 

Figure 6.5. The model updating results from the experimental data for lateroflexion. The quasi-static stiffness estimates of the four intervertebral discs are represented by the black squares and the injected levels are marked by the squares with the black face colour. The grey squares represent the dynamic stiffness estimates and the circles the estimated masses of the vertebrae. Vertebral height is indicated by the dots, #1 is the estimated vertebral height, the other heights were measured on MRI. The data file of goat 1 was corrupted, therefore this graph is missing.
Table 6.1 gives the mean squared errors (MSE) between quasi-static and dynamic stiffness estimates per level. The MSE was lower for lateroflexion than for flexion-extension, except for level 3. The worst fit was found for flexion-extension level 1, the best for lateroflexion level 2. The data of goat 1 for lateroflexion were corrupted; therefore these results are missing in the graphs and in the table.

<table>
<thead>
<tr>
<th></th>
<th>Flexion-extension</th>
<th>Lateroflexion</th>
</tr>
</thead>
<tbody>
<tr>
<td>All levels</td>
<td>71.52</td>
<td>40.07</td>
</tr>
<tr>
<td>Level 1 (free end)</td>
<td>174.50</td>
<td>42.24</td>
</tr>
<tr>
<td>Level 2</td>
<td>13.28</td>
<td>2.10</td>
</tr>
<tr>
<td>Level 3</td>
<td>21.05</td>
<td>61.70</td>
</tr>
<tr>
<td>Level 4 (fixed end)</td>
<td>77.23</td>
<td>54.34</td>
</tr>
</tbody>
</table>

The circles in Figure 6.4 and Figure 6.5 represent the estimated masses. A remarkable finding is that the mass of the top vertebra was always estimated to be higher than the mass of the other vertebrae. Although the “real’ masses of the vertebrae are unknown, it is not likely that the top vertebra had the highest mass since this vertebra was cut in half to free it from the embedding.

The MAC values and the differences between the experimental eigenfrequencies and the model eigenfrequencies for the first three modes for the six goats are shown in Figure 6.6. The differences between the eigenfrequencies was normalized to the values for the experimental eigenfrequencies per mode, in order for the values to be comparable. The graph shows that the MAC was highest for the first mode and mostly lowest in mode 3. The difference between the experimental and the analytical eigenfrequencies was highest for the third mode, and lowest for the second mode.

**DISCUSSION**

The aim of this study was to investigate whether model updating can be used for vibration-based damage identification in the lumbar spine. Since cut-off stiffness values that discriminate between healthy and degenerated joints are not available, the method does not strive for an accurate measure of absolute stiffness of a single
Figure 6.6. The MAC-values (left three stems) and the normalized differences between the experimental eigenfrequencies ($\times 100$) and the model eigenfrequencies (right three stems) for modes 1 to 3 for the six goats. The top half of each graph contains the values for flexion-extension (FE), the bottom half for lateroflexion (LF). Also here the data for lateroflexion for goat 1 is missing.
joint, but aims to identify the intervertebral joints with a largely different bending stiffness than adjacent segments. Since large differences in stiffness between adjacent segments might be the cause of spinal disorders, positive or negative outliers in the stiffness matrix might have clinical meaning.

Since it was previously shown that not only artificially induced damage to the intervertebral disc can be detected by vibration analysis [100], but that also intervertebral degeneration caused by physiological processes can be measured [106], this study performed vibration measurements on ‘naturally’ degenerated multisegmental specimens. The specimens were obtained from lumbar goat spines to ensure that the specimens also contained healthy intervertebral discs for comparison. This would have been less likely when human spines were used, due to the unavailability of young donor material. Since it is difficult to obtain mild degeneration in cadaverous tissue without the risk of unintended deterioration of the whole tissue, a live animal model was used in which mild degeneration was created by injecting lumbar intervertebral discs with chondroitinase-ABC. Previous research showed that 12 weeks after the injection mild degeneration had occurred [118], as this study aimed for. Unfortunately, as the quasi-static mechanical tests showed, not all injected levels showed decreased stiffness values compared to levels that were not injected. This could indicate that the injections were ineffective. On the other hand, it could be a result of pre-existing variance of segmental stiffness in the spines. However, due to the fact that live animals were used, it was not possible to perform pre-injection stiffness tests. Therefore, it was decided not to use the knowledge of positive or negative for injection for comparison with the estimated stiffness after model updating, but to compare the estimated stiffness values to the static stiffness values, thereby assuming that no measurement error occurred during the assessment of the static stiffness.

The results showed that the correlation between the quasi-static stiffness estimates and the dynamic stiffness estimates was low. Identification of the damaged levels by selecting the two lowest values in the stiffness matrix did not point to the same levels as the two lowest quasi-static stiffness estimates. This implies that vibration-based damage identification using these experimental data and this analytical vibration model would not be possible. However, close scrutiny of the data in Figures 6.3-6.5 suggests that something was amiss with the estimation of vertebral mass. In both the
simulated data, as in the experimental data, the estimates of vertebral mass appear to have shifted easily during model updating. Although the true mass of the vertebrae was not known, it is unlikely first, that the estimation of mass from vertebral height was so far off to necessitate such shifts and second, that the differences in mass between adjacent vertebrae would be so large. From a biomechanical perspective it does not seem strange that the mass at the fixed end shifted; to obtain a minor change in the mode shapes and in the eigenfrequencies, a major change in mass is necessary. Due to the close proximity of the mass to the fixed end, it has only a short moment arm and thus this mass has a small effect on the eigenfrequencies and the mode shapes. On the other hand; large changes to the eigenfrequencies and the mode shapes can be achieved by changing the mass at the free end. Therefore, future research into vibration-based damage identification using model-updating procedures should obtain accurate values for the mass of the vertebrae, so that the mass can be omitted from model updating. As previously shown, this can be achieved via modelling of the vertebrae based on quantitative computed tomography images [117].

The correlation between the quasi-static and dynamic stiffness estimates might also be low due to flaws in the experimental data. Several sources of error affect vibration experiments. By checking the coherence between vibration input and output, it was verified that the quality of the measurement was sufficient, and no errors were introduced for example by a loose contact between shaker and vertebra. Second, vibration analysis assumes a linear system. Although it is known that the spine is a highly non-linear structure, it was previously shown that at small deflections (within the neutral zone) it behaves linearly [100]. Finally, deflections of the vertebrae are preferably measured at the point of maximum deflection to ensure optimal signal-to-noise ratios and not at nodal points where the deflection is zero. Ideally, the vibration response of a structure is analysed beforehand using finite element models to assess the optimal location of the measurement points. However, in this study the deflections were not studied before the actual measurement trials took place and suboptimal placement of measurement points may therefore have occurred. Theoretically, and in ideal beam structures, consecutive eigenfrequencies are spaced at fixed intervals:

\[
f_1 = \frac{3.52}{2\pi} \sqrt{A}, \ f_2 = \frac{22.4}{2\pi} \sqrt{A}, \ f_3 = \frac{61.7}{2\pi} \sqrt{A}
\]  

(6.6)
where \( f_1, f_2 \) etc are the eigenfrequencies, and \( A \) represents the expression to calculate the eigenfrequencies [119]. Although the spine obviously is not an ideal beam structure, the experimental mode shapes did resemble the modal behaviour of cantilever beams to some extent. By comparison, the intervals between the experimental eigenfrequencies suggest that some frequencies and corresponding mode shapes may have been missed. This might explain why the third mode showed large differences in estimated and experimental eigenfrequencies (Figure 6.6). At higher order modes, the distinction between pure bending modes and modes in which also some coupling with other motion directions takes place is more difficult, possibly causing misinterpretation of modal data and fitting analytical modes to the “wrong” experimental modes. Future studies should assess the modal characteristics of the model to decide on optimal placement of measurement points and to prevent “missing” modes and corresponding eigenfrequencies.

Finally, the correlation between quasi-static and dynamic stiffness estimates might be low because the present analytical vibration model was not able to describe the modal characteristics of the specimens with sufficient precision. The decision to model only rotational degrees of freedom might to some extent have hampered the fit between the analytical and experimental data. However, the results from an unpublished pilot study revealed that overestimated translational stiffness has no influence on the eigenfrequencies and modes shapes, in contrast to underestimated translational stiffness. Therefore, the translation stiffness in the present model was set at an infinitely high value to prevent it from causing differences between experimental and analytical modes. Furthermore, no damping was assumed. Although probably different damping mechanisms are present in the spine [89], their influence on the eigenfrequencies is expected to have been sufficiently small not to hamper the fitting procedure. Nevertheless, damping and translational stiffness require more research to definitely exclude their influence on the estimation of the stiffness matrix and both are interesting parameters for further investigations.

**CONCLUSION**

The results from this study do not allow firm statements about the ability of vibration-based damage identification in the lumbar spine without knowledge of healthy baseline information on modal properties. However, this study does show a probable
approach for such measurement method and it shows plausible directions for further improvement of the methodology.

ACKNOWLEDGEMENTS

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