Summary

The main purpose of derivative estimation, a field within Monte Carlo simulation, is the development of efficient estimators for derivatives with regard to parameters present in a discrete-event stochastic model. To clarify, a discrete event stochastic model is a (complex) measurable mapping, which is typically defined through a simple sample performance function of a finite collection of random variables or implicitly defined via a stochastically recursive sequence. The parameters within the model are either *intrinsic*, located in one or more random variables, or *extrinsic*, being a part of the model though only induced by the mapping describing the interaction of the basic input random variables.

Discrete-event stochastic models represent a wide variety of applications. Standard examples of these models are waiting times of customers in a (first-in-first-out) single-server queue, and the payoff function of an European financial option. The single-server queue can be extended to a queuing network representing a call-center or logistics network. Correspondingly, we can extend the pricing model for the option to evaluate the value of a portfolio consisting of assets and derivatives.

Derivative estimation is of importance in the analysis of discrete-event stochastic models. For example, acquiring the derivatives and determining the resulting estimator is an immediate method for model validation. Indeed, if the model is highly sensitive with respect to some model parameter the true value of which can only be obtained by statistical estimation, then the model becomes unreliable. On the other hand, if the sensitivity of the model with respect to a parameter is low, this indicates that the parameter might be superfluous and a simpler model could be analysed instead. Specifically, obtaining parameter sensitivities of financial options provides us with Monte Carlo estimates of the 'Greeks'. In addition, sensitivity estimation provides the means for evaluating the derivative expressions in stochastic optimisation. For instance, the Robbins-Monro algorithm can find the optimal fund allocation for a single-period hedge in a portfolio.

There are three main approaches to derivative estimation: the Finite Difference (FD) method; pathwise methods which is primarily Infinitesimal Perturbation Analysis (IPA), or Smoothed Perturbation Analysis (SPA); and distributional differentiation, namely, Score Function method (SF); and Measure-Valued Differentiation (MVD). The FD estimator is commonly used as it is simple to attain an estimate. However, FD produces biased estimates, can in instances yield a sub-optimal rate of convergence, and there is always a more precise estimation method. On the other hand, both pathwise and distributional approaches to

sensitivity estimation yield unbiased estimators. Perturbation analysis requires the random variable to be a measurable mapping of the parameter of interest. For IPA, if permissible, the derivative is obtained directly from the random variable. SPA smoothing out discontinuities by conditioning before obtaining the pathwise derivative. In contrast, distributional approaches differentiate the underlying distribution, where the parameter of interest is assumed to be a parameter of a distribution. The resulting SF estimator method provides a distribution-based augmentation to the stochastic model, whereas MVD rewrites the derivative as a difference of two distributions expressing the derivative as a stochastic finite difference. A brief introduction into derivative estimation is provided in Chapter 1

This thesis identifies and addresses two limitations within derivative estimation. The first limitation is that in derivative estimation praxis, estimators for parameter derivatives are only obtained for expectations or probabilities, where a probability is treated as an expectation with regard to an indicator mapping. The second limitation is that parameter sensitivities with regard to extrinsic parameters are difficult to obtain, where an example of an extrinsic parameter is the barrier level of an European option.

The first limitation is addressed in the first two chapters. In Part I of Chapter 2 a systematic investigation of IPA, SF, and MVD derivative estimators is provided for performance measures pertaining ranked-data, namely order statistics and quantiles. Two estimators for both of the distributional approaches are developed, a statistical analysis is provided, and a series of numerical examples is conducted to ascertain the performance of the three methods. In most instances IPA is the best approach, and MVD is the best distributional alternative. This chapter also provides an almost sure version of the strong consistency proof, improving upon results known in the literature.

In Chapter 3 a theoretical analysis of the performance of the two MVD ranked-data derivative estimators is provided.

In Chapter 4, we propose an alternative class of single-run quantile sensitivity estimators for i.i.d. random variables that enjoys the modularity of the distributional sensitivity estimation approach. In the most general setting, we again use MVD to estimate the parameter derivative of the empirical cumulative distribution function.

In Chapter 5 the second limitation is addressed, developing derivative estimators for the price barrier-type stock options with respect to the barrier level. This is achieved via SPA. Using an elegant choice of sample performance function, the barrier level sensitivity for the European Barrier and the Step/Parasian option is conducted, assuming a continuous monitoring of the stock price. Secondly, for discrete monitoring of the price, the barrier level sensitivity for the Parisian option is obtained. The principal aspect for this sensitivity is the notion of the critical event and the counting of these critical events along the price path. The Black-Scholes process is used to model the uncertain price processes.

The appendices of the thesis provides some technical results.