Chapter 3

Asymmetric Nash Solutions in Trans-boundary River Sharing Problems

3.1 Introduction

As mentioned in Chapter 1, the main issues in trans-boundary river basins are scarcity of water and externalities from upstream to downstream. For instance, excessive water consumption of upstream countries might deprive the right of water consumption for downstream countries. Another prominent issue is that upstream countries could pollute the trans-boundary river carelessly without taking into account the downstream countries' benefits. Unfortunately, the property rights of water in trans-boundary river basins are not properly defined. Therefore, some particular characteristics of the river sharing problem such as the externalities of pollution from upstream to downstream and the absence of clearly defined property rights in international river situations have drawn interests from researchers to study the river sharing problem.

Giannias and Lekakis (1996) and Kilgour and Dinar (2001) analyze the river sharing problem between two or more countries along an international river that is linear, i.e., a river where agents are located subsequently from upstream to downstream. The model in the first reference distinguishes between upstream and downstream, is deterministic and has both water quantity and water quality. The second reference analyzes a stochastic model of water quantity among several countries. Both studies characterize the unique allocation that maximizes utilitarian welfare. Ambec and Sprumont (2002) mark the start of embedding legal principles from International Water Law in the river sharing problem. They translate the legal principles of Absolute Territorial Sovereignty (hereafter, ATS) and Unlimited Territorial Integrity (hereafter, UTI) into their model. ATS is applied to every group of agents and imposes conditions of core stability. UTI is also applied at the group level, but it is an aspirational approach. It is impossible that distinct groups of countries can simultaneously achieve their group aspiration levels. As appropriate requirements, Ambec and Sprumont (2002) propose group ATS and that no group attains a welfare above its group aspiration level. For a linear river with insatiable agents, they show that the downstream incremental solution is the only welfare distribution that satisfies these two requirements. Ambec and Ehlers (2008) generalize this result by allowing for agents with a satiation point. In van den Brink et al. (2012), other legal principles and more general river geographies are considered.

International Water Law states that countries should mutually agree on sharing the river through negotiations. For that reason, we approach the river sharing problem from a bargaining perspective. Ambec and Ehlers (2008) show that the downstream incremental solution can be implemented as the unique outcome of a sequential bargaining process. In this process, lexicographical priority is given who proposes in the order from downstream users to upstream users. A proposal is a feasible allocation of water and monetary transfers among the proposer and all of his upstream users. If a proposed allocation is rejected, its proposer is committed to leave the bargaining and restricted to use his own local water resource. The remaining players continue the bargaining process. Lexicographical priority is an extreme form of bargaining power, upstream users have less flexibility in proposing than downstream users and the commitment to be excluded (or to refrain) from future bargaining is a too strong assumption. Clearly, there is a need for a more realistic bargaining perspective.

The literature on bargaining does provide a general theory based upon arbitrary distributions of bargaining power. Because International Water Law states that countries should mutually agree on the water allocation, unanimity among these countries is required. This makes the asymmetric Nash bargaining solution (hereafter, ANBS) a natural candidate for analyzing the river sharing problem. The ANBS has been axiomatized in e.g., Kalai (1977), Kaneko (1980) and Herrero (1989), and it is supported by strategic bargaining models in e.g., Herrero (1989), Miyakawa (2006), Laruelle and Valenciano (2008) and Herings and Predtetchinski (2010). Application of the ANBS to the river sharing problem with only two agents, an upstream and a downstream agent, can be found in e.g., Houba (2008) and Houba et al. (2013). In this chapter, we generalize this approach to a general river geography with multiple agents.

As noticed in Houba (2008), legal principles not only restrict the negotiations to unanimity bargaining, but also have implications for the countries' strategic possibilities as long as they do not cooperate, i.e., the disagreement outcome. In this chapter, we apply the ATS principle and the UTI principle as the guiding principles for individual countries in case of disagreement. For UTI, we discuss two interpretations: a strict interpretation in which only the most downstream country is allowed to use water; and an interpretation in which each country claims UTI. The ATS and strict UTI imply different disagreement outcomes that are both feasible. Under the second UTI interpretation the vector of individual aspiration levels under disagreement is infeasible and yields a utopia point. And agreement can only be reached if the countries are willing to compromise on these levels. In Mariotti and Villar (2005), the Nash rationing solution is given and axiomatized to study compromise situations in which unanimity is required. Their solution is symmetric, possibly multi-valued and always contains the maximizers of a modified Nash product over the Pareto frontier. For situations with transferable utility, the Nash rationing solution is unique and coincides with the unique maximizer. In this chapter, we propose a modification of the Nash rationing solution to allow for asymmetries.

We model multiple agents along a general river structure that is expressed by a geography matrix and who have access to limited local resources as in e.g., van den Brink et al. (2012) and Ansink and Houba (2012). Each agent has quasi-linear preferences over water and money, where the use of water yields a net benefit being the difference of the benefit of water use and the cost of water extraction. The extraction costs depend upon the amount of the available water and the amount of extraction. Maximizing the net benefits yields a satiation point that depends on the available amount of water and this generalizes Ambec and Ehlers (2008) in which the satiation point is fixed.

Compared to the current literature, this chapter makes several contributions.

First, translating the ATS principle and the two interpretations of the UTI principle into individual levels of welfare under disagreement, we arrive at three different cases. In the ATS and the strict UTI case the vector of disagreement levels is feasible and serves as the different disagreement outcomes in the ANBS. Different legal principles define different property rights, and therefore the associated ANBS outcomes will also be different and allow an interpretation in terms of a shift in property rights. We also characterize the negotiated welfare distribution.

Second, we revisit the political economy of establishing property rights through the different legal principles of ATS and strict UTI. Rational agents are forward looking and are not interested in these legal principles as such, but rather how these affect the negotiations and the final agreement. Whatever the political process in which agents decide on property rights before negotiating joint river management, each agent tries to invoke the legal principle that serves his best interest. According to common intuition, in the two-agent case the most downstream agent always prefers the strict UTI principle, because it gives him the right to claim all water inflows along the river, and the upstream agent prefers ATS, because it gives him the right to claim his own water inflow. For the n-agent case, our analysis confirms this common intuition for the most downstream agent, while at least one of the other agents prefers the ATS principle. However, other than the common intuition for the two-agent case, in the n-agent case other agents besides the most downstream agent may prefer the strict UTI principle above the ATS principle, which reverses the common intuition. It appears that strong bargainers amongst the other agents may prefer the strict UTI principle.

Third, since the individual aspiration levels are infeasible under the strict interpretation of UTI, we apply to this situation the asymmetric Nash rationing solution (hereafter, ANRS). The asymmetric weights in this solution cannot be interpreted as bargaining weights, because a larger weight yields a lower welfare. Instead, it is intuitive to interpret these weights as responsibility weights. Mathematically the ANRS has many similarities with the ANBS, but we show by means of an example that at the ANRS some agents might receive a welfare that is below what could be obtained by blocking agreement, i.e., refrain to use water and nonparticipation in the negotiations, which would yield zero payoffs. To avoid such outcomes, we propose to add participation constraints to the asymmetric Nash rationing problem. These constraints can also be justified by modelling a ratification process that takes place after the negotiations are concluded.

Fourth, we show that the maximization of the (modified) Nash products is separable into two subproblems: the efficient water allocation that maximizes utilitarian welfare and that can be related to the geography matrix; and the monetary transfer associated with the bargaining weight. In order to derive general formulas that are also applicable if the consequences of other legal principles from International Water Law are studied, we analyze the ANBS, respectively ANRS, under unspecified disagreement (utopia) points.

This chapter is based on Houba et al. (2014b) and is organized as follows. In Section 3.2, we specify the river sharing model and introduce the general river geography. Then, in Section 3.3, we discuss several legal principles and translate them into either a disagreement point or a utopia point. In Section 3.4, we first analyze the ANBS for unspecified disagreement points and show that the maximization is separable into two subproblems, before analyzing specific disagreement points associated with the mentioned legal principles. In Section 3.5, the individual aspiration levels are analyzed from a Nash rationing perspective. Section 3.6 contains two numerical examples and Section 3.7 concludes this chapter.

3.2 Model specification

We consider a river that flows through a finite set of locations, for instance cities, agriculture communities, industrial facilities or countries, at which water is extracted from the river. These locations are called agents and the set of agents is denoted by $N = \{1, 2, ..., n\}$, where $n \geq 2$ is the number of agents.

The river geography is represented by a graph with the agents as its nodes. For the ease of exposition we only consider directed trees, where the root of the tree represents the most downstream agent, numbered by n, and arcs are directed to the root. This collection of possible river geographies includes the linear river in Ambec and Sprumont (2002) and rivers that originate at multiple springs, which merge together downwards into a single stream, as considered in e.g., van den Brink et al. (2012) and Ansink and Houba (2012). The last reference discusses extensions that allow for rivers with a delta, multiple users per location and enforceable legal entitlements, which would apply to interstate transfers as in e.g., Heintzelman (2010). All of our results can be adjusted in a straightforward manner because they do not affect the mathematical structure of the model.

Every agent located downstream to agent i is said to be a successor of i and we denote the set of all successors of i by S^i. Because n is the root of the tree, we have $S^n = \emptyset$ and $S^i \neq \emptyset$ for all $i \neq n$. Similarly, the set P^i denotes the set of all predecessors of i located upstream of i along the river. An agent i has $P^i = \emptyset$ if and only if i is located at a spring or source of the river. Furthermore, we notice that $P^n = N \setminus \{n\}$.

The natural water inflow, possibly zero, at the territory of agent i, $i \in N$, is denoted by e_i and the amount of water used by agent i is denoted by x_i. Furthermore, all predecessors of agent i could potentially transfer water to i, whereas i could possibly transfer water to his successors. The amount of water available for agent i is given by $f_i = e_i + \sum_{j \in P^i}(e_j - x_j)$, which consists of his own local water resource e_i plus the inflow of water that his predecessors transfer to i. Since water only flows from upstream to downstream and inflow at successors of i can not be allocated to i, the water use of agent i is constrained by $x_i \leq f_i$. In the sequel we denote $e = (e_1, \ldots, e_n)^\top \in \mathbb{R}^n_+$ as the vector of natural inflows, $x = (x_1, \ldots, x_n)^\top \in \mathbb{R}^n_+$ as the vector of water uses and $f = (f_1, \ldots, f_n)^\top \in \mathbb{R}^n_+$ as the vector of constraints.

Because it might be convenient to work in matrix notation, we model the river geography by the $n \times n$ matrix R with components R_{ji} given by $R_{ji} = 1$ if $j \in S^i \cup \{i\}$, and $R_{ji} = 0$ otherwise, which follows Ansink and Houba (2012). Using this allows us to rewrite the vector f of available water and all water constraints $x \leq f$ as

$$f = e + (R - I)(e - x), \qquad \text{respectively,} \quad Rx \leq Re. \tag{3.1}$$

Note that (3.1) specifies a non-empty and convex set in the (x, f)-space. The next example illustrates our notation.

Example 3.1. Consider a river with two springs at locations 1 and 2 that merge together at location 3 before it flows through location 4. The river geography is represented by

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

For instance, since water from location 1 can be used in locations 1, 3 and 4, the first column of R is given by $(1\ 0\ 1\ 1)^\top$. The (in)qualities for water inflows and feasibilities (3.1) imply the (in)equalities

$$\begin{bmatrix} f_1 = e_1 \\ f_2 = e_2 \\ f_3 = e_3 + \sum_{j=1}^{2} (e_j - x_j) \\ f_4 = e_4 + \sum_{j=1}^{3} (e_j - x_j) \end{bmatrix} \text{ and } \begin{bmatrix} x_1 \leq e_1 \\ x_2 \leq e_2 \\ \sum_{j=1}^{3} x_j \leq \sum_{j=1}^{3} e_j \\ \sum_{j=1}^{4} x_j \leq \sum_{j=1}^{4} e_j \end{bmatrix}.$$

□

By the tree structure of the river geography, we have the following two properties on the matrix R.

Property 3.1.
(i) If $R_{ji} = 1$, then $R_{ij} = 0$.
(ii) If $R_{ji} = 1$ and $R_{ki} = 1$, then either $R_{kj} = 1$ or $R_{jk} = 1$.

The first property reflects that if water can flow from agent i to $j \neq i$, then it is impossible that the water flows from j to i. This rules out locations that have a local common pool, for instance situations in which the river is the common border between two countries. The second property reflects that we don't allow that the river splits into a delta, i.e., each agent $i \neq n$ has precisely one downstream neighbor. This is ruled out, because otherwise additional information is needed regarding how the water flow divides amongst different branches, which may depend on geographical factors, for instance the differences in altitude along the several branches, as well as the flow level at the point of splitting.

Given the constraints $Rx \leq Re$ on the use of water, each agent along the river chooses an amount x_i of water use for industrial production, residential use, irrigation etc. An amount x_i yields benefits of the water use and costs of water extraction for each agent. Agent i's cost depends upon the amount of water extraction x_i and the available water f_i.

Assumption 3.1. *Agent $i \in N$ has a benefit function $b_i \colon \mathbb{R}_+ \to \mathbb{R}_+$ with the property that $b_i^{'} > 0$, $b_i^{''} < 0$ and $b_i(0) = 0$.*

Assumption 3.2. *Agent $i \in N$ has a cost function $c_i \colon \mathbb{R}_+^2 \to \mathbb{R}_+$ with the property that $\frac{\partial c_i}{\partial f_i} < 0$, $\frac{\partial c_i}{\partial x_i} > 0$, $\frac{\partial^2 c_i}{\partial x_i^2} > 0$, and $c_i(f_i, 0) = 0$ for all $f_i \geq 0$.*

The inequality $\frac{\partial c_i}{\partial f_i} < 0$ means that water use of upstream agents generates negative externalities for downstream agents. The costs of extraction are decreasing in the amount of available water, i.e., more use of water by the predecessors of agent i and thus a decrease of f_i results in higher extraction costs for the same amount x_i. So a decrease in f_i yields an upward shift of the entire cost function, except at $x_i = 0$. The conditions on the first and second derivative of c_i to the extraction x_i of water imply that the cost function is convex in agent i's own water use. Finally, we assume that zero extraction yields zero costs, independent of f_i. This assumption is merely made for convenience in Section 3.5 where it implies zero utility from inaction or nonparticipation in an agreement. None of our other results in this chapter depends upon our assumption that zero extraction yields zero costs.

We further assume that utility is transferable in the sense that agents are able to transfer utility to each other by making monetary transfers. The monetary transfer to agent i is equal to $t_i \in \mathbb{R}$. A positive transfer $t_i > 0$ means that agent i receives money and $t_i < 0$ means that agent i has to pay $|t_i|$. A monetary transfer scheme is a vector $t = (t_1, \ldots, t_n) \in \mathbb{R}^n$ such that there is no financial deficit: $\sum_{i=1}^n t_i \leq 0$.

The utility of agent i depends on x_i, f_i and t_i and is given by the quasi-linear utility function

$$u_i(f_i, x_i, t_i) = b_i(x_i) - c_i(f_i, x_i) + t_i,$$

where $b_i(x_i) - c_i(f_i, x_i)$ is the net benefit of the water use x_i at f_i. Notice that by our assumptions, the net benefit $b_i(0) - c_i(f_i, 0)$ of inaction is equal to zero for every $f_i \geq 0$. Maximizing the net benefit yields the first-order condition

$$\frac{\partial b_i}{\partial x_i} - \frac{\partial c_i}{\partial x_i} = 0.$$

In case this equation has a solution, then it is the satiation point of agent i and this satiation point, say $s_i(f_i)$, depends on f_i.

In summary, the river sharing model is fully represented by the quadruple (N, R, u, e), where N denotes the set of agents, R is the river geography, u is the collection of utility functions $\{u_i\}_{i \in N}$ and e is the vector of local water resources. In the remainder of this chapter, we assume that each agent in this model is a rational utility maximizer and that all benefit functions, cost functions and water resources are common knowledge.

The maximal utilitarian welfare is obtained by maximizing the sum of all net benefits, so by solving the optimization problem

$$\max_{x, f \geq 0} \sum_{i=1}^{n} u_i(f_i, x_i, 0) \text{ s.t. (in)-equalities (3.1).} \tag{3.2}$$

The global maximum value, denoted by w, is unique. Since individual utilities are transferable through the monetary transfers, the utility possibility set is given by $U = \{u \in$

$\mathbb{R}^n | \sum_{i=1}^n u_i \leq w\}$, see e.g., Mas-Colell et al. (1995), p.325. Therefore, the global maximum value w at a solution of problem (3.2) describes what can be achieved in the river sharing problem (N, R, u, e) in terms of welfare. Note that without additional assumptions on the benefit and cost functions, optimization program (3.2) may admit multiple maximizers to support w. Since the uniqueness of w drives our analysis, for ease of discussion and to relieve the notational burden, in the sequel of this chapter we restrict ourselves to cases with a unique maximizer. In fact, since (3.1) specifies a convex set in the (x, f) space, uniqueness of the maximizer is guaranteed if all cost functions $c_i(f_i, x_i)$ are strictly convex in (f, x) in addition to Assumption 3.1 and 3.2. We denote the unique maximizer by (x^{UW}, f^{UW}).

Furthermore, it might be that $x_i^{UW} = 0$ for all $i < n$, i.e., welfare is maximized at zero extraction by all agents $1, \ldots, n-1$. However this seems to be unrealistic in practice and also for ease of analysis we exclude this case.

Assumption 3.3. *For at least one agent $i = 1, \ldots, n-1$, it holds that $x_i^{UW} > 0$.*

Our framework captures some of the influential models of the river sharing problem. Ambec and Ehlers (2008) assume that the (net) benefit function only depends on x_i, is strictly concave and might have a satiation point s_i. Under our assumptions, the concavity of the benefit function b_i and the convexity of the cost function c_i in x_i yield a concave net benefit function in x_i that might have a satiation point. Since the cost function c_i depends on the available water resources f_i, the satiation point $s_i(f_i)$ also depends upon f_i, so our model generalizes Ambec and Ehlers (2008).

Our model can also be interpreted in terms of pollution externalities. For instance, van der Laan and Moes (2012) incorporate pollution in the benefit function and the cost function. In their model, an agent's cost function depends on accumulated own pollution and the pollution of all his upstream agents. If we treat water use as positively correlated with pollution and some convex function $\hat{c}_i \colon \mathbb{R}_+ \to \mathbb{R}_+$ defines the cost function $c_i(f_i, x_i) = \hat{c}_i(f_i - x_i)$, then c_i depends upon accumulated water use $\sum_{j \in P^i \cup \{i\}} x_j$, which is identical as in van der Laan and Moes (2012). Our cost function generalizes this by allowing for asymmetric effects between upstream pollution and own pollution, but makes the additional assumption that the costs are zero when the own pollution is zero.

3.3 Legal principles defining property rights

Legal principles from International Water Law have spurred a new emerging literature in the river sharing problem following Ambec and Sprumont (2002). In this section, we first discuss several legal principles that define different property rights regimes for international rivers, and then, translate these into our framework. Two dominant principles in International Water Law are the principle of 'reasonable and equitable water use' and the principle of 'no significant harm', see e.g., Salman (2007). In the rhetoric of real negotiations, the nuance underlying these principles often vanishes and the extreme versions of these principles appear instead: the principle of Absolute Territorial Sovereignty (hereafter, ATS) and the principle of Unlimited Territorial Integrity (hereafter, UTI), which are also more convenient for modelling. Both of these extreme principles will be discussed in a separate section.

3.3.1 Absolute Territorial Sovereignty

As introduced in Chapter 1, the ATS principle, also known as the Harmon doctrine, states that a country has absolute sovereignty over the area of any river basin on its territory: it may freely decide how much water to use of the water flowing within its borders but cannot claim the continued and uninterrupted flow from upper basin countries. Alternatively, ATS also describes situations of 'laissez-faire' regulation or anarchy among water users.

In our framework, every agent i has the property rights over his own local water resource and inflow from upstream under ATS. In this situation, agent i can freely consume f_i. This includes his own local inflow e_i and all the unused water from his predecessors without the obligation to pay any monetary compensation. Starting from the agents i with $P^i = \emptyset$, we can recursively solve for the inflows f_i^{ATS} that will result when all agents maximize their own net benefits by

$$f_i^{ATS} = e_i + \sum\nolimits_{j \in P^i} (e_j - x_j^{ATS}), \quad \text{where } x_i^{ATS} = \arg\max_{x_i} u_i(f_i^{ATS}, x_i, 0), \text{ s.t. } x_i \leq f_i^{ATS}, \tag{3.3}$$

where Assumption 3.1 and 3.2 imply uniqueness. The disagreement pair (x^{ATS}, f^{ATS}) differs from the unique maximizer (x^{UW}, f^{UW}) of (3.2), because the ATS does not internalize the externalities of water use x_i on downstream's benefits of water use nor the externalities of f_i on the cost of extraction. So, there exists a group of at least two agents who can beneficially trade water to increase utilitarian welfare.

In negotiations, the net benefits associated with these water uses and inflows specify the disagreement utilities given by $d_i^{ATS} = u_i(f_i^{ATS}, x_i^{ATS}, 0)$, $i = 1, \ldots, n$. Since every agent can guarantee himself a zero net benefit by a zero extraction of water, it holds that $d_i^{ATS} \geq 0$ for all i. The following result shows that the disagreement point under the ATS principle yields strictly less welfare than the maximal utilitarian welfare w as defined in (3.2). The proofs of the results in this chapter are deferred to the appendix of this chapter.

Proposition 3.1. *In the river model (N, R, u, e) it holds that $\sum_{i=1}^{n} d_i^{ATS} < w$.*

A final remark concerns the econometric estimation of all benefit functions, all cost functions and the vector $d^{ATS} = (d_1^{ATS}, \ldots, d_n^{ATS})$ of disagreement utilities. Such estimation is possible in applications where it is a priori known that all agents conform to ATS and time-series data for x, f, e and values for costs and benefits are observed. Furthermore, this estimation will be performed on (3.3). Such estimation has been performed in e.g., Fernandez (2002) and the references therein. Due to a lack of data, Houba et al. (2013) resort to calibration in their static annual two-season model.

3.3.2 Unlimited Territorial Integrity

As introduced in Chapter 1, the UTI principle states that a country has the right to demand the natural flow of an international river into its territory that is undiminished in quantity and unchanged in quality by the upstream countries. Incorporating it into our framework, an upstream agent is only allowed to consume water if he has the explicit consent of all his downstream agents. As recognized in e.g., McCaffrey (1996, 2001), when all agents invoke the UTI principle, UTI itself becomes self-contradictory. In the case of one upstream and one downstream agent, when both agents invoke the UTI principle, the local water resource on the territory of the upstream agent is claimed by both, leading to inconsistency. In the following discussion, we consider two interpretations of the UTI principle: according to the first strict interpretation, only the most downstream agent may claim all water inflows, in the second interpretation the UTI principle is invoked by all agents.

3.3.2.1 Strict UTI

The UTI principle clearly favors downstream agents over upstream agents. Hence, in practice, the UTI principle has often been invoked by downstream agents. In this section, we take the most restrictive case under the UTI principle, namely that only the most downstream agent may claim all the water of the river and can restrict all his predecessors to zero extraction as long as no agreement has been reached.

Formally, the disagreement utilities are obtained as follows. For every agent $i = 1, 2, ..., n-1$ we have that under disagreement $x_i^{UTI} = 0$, thus the disagreement flows are given by $f_i^{UTI} = e_i + \sum_{j \in P^i} e_j$. For the most downstream agent n, we have that $f_n^{UTI} = \sum_{j \in N} e_j$ and x_n^{UTI} is the solution to the maximization problem

$$x_n^{UTI} = \arg\max_{x_n} b_n(x_n) - c_n(f_n^{UTI}, x_n), \text{ s.t. } x_n \leq f_n^{UTI}, \tag{3.4}$$

where uniqueness is guaranteed by Assumption 3.1 and 3.2. This gives the disagreement utilities $d_i^{UTI} = u_i(f_i^{UTI}, 0, 0) = 0$ for agents $i = 1, \ldots, n-1$ and $d_n^{UTI} = u_n(f_n^{UTI}, x_n^{UTI}, 0) = u_n(\sum_{j \in N} e_j, x_n^{UTI}, 0) > 0$. The disagreement utilities under the strict UTI principle yield strictly less welfare than the maximal utilitarian welfare w. We state the following result without proof,[1] and the strict inequality is implied by Assumption 3.3 that (x^{UW}, f^{UW}) and (x^{UTI}, f^{UTI}) are unequal.

Proposition 3.2. *In the river model (N, R, u, e) it holds that $\sum_{i=1}^{n} d_i^{UTI} < w$.*

Since $\sum_{i=1}^{n-1} d_i^{ATS} \geq 0 = \sum_{i=1}^{n-1} d_i^{UTI}$ and $d_n^{ATS} < d_n^{UTI}$, agent n has a strictly better bargaining position under UTI than under ATS and the other agents have a reverse order with respect to their bargaining positions. However, the final utility also depends on the net surplus (i.e., the welfare from cooperation minus the sum of disagreement utilities). In the next section, we will clarify how these two effects determine the final utility for each agent.

Under strict UTI, it might happen that $x_n^{UTI} < f_n^{UTI}$, so $x_n^{UTI} = s_n(f_n^{UTI}) = s_n(\sum_{j \in N} e_j)$, i.e., agent n extracts his satiation level of water and leaves some of the total flow $\sum_{j \in N} e_j$ unused. This raises the issue whether agent n's predecessors should be allowed to use water. The answer is negative. An intuitive explanation is that any water use by the upstream agents generates negative externalities on the most downstream agent in the sense of decreasing his water inflow, hence increasing his extraction cost. Indeed, denoting

[1] The proof is similar to the proof of Proposition 3.1.

$\hat{x}_n$ as the solution of $\max_{x_n \le f_n^{UTI} - x_{-n}} b_n(x_n) - c_n(f_n^{UTI} - x_{-n}, x_n)$, where $x_{-n} > 0$ denotes the aggregate water use amount by the predecessors of agent n, it follows that

$$b_n(\hat{x}_n) - c_n(f_n^{UTI} - x_{-n}, \hat{x}_n) < b_n(\hat{x}_n) - c_n(f_n^{UTI}, \hat{x}_n) \le$$

$$\max_{x_n \le f_n^{UTI}} b_n(x_n) - c_n(f_n^{UTI}, x_n) = b_n(x_n^{UTI}) - c_n(f_n^{UTI}, x_n^{UTI}),$$

and the strict inequality comes from the fact that $c_n(f_n, x_n)$ decreases when the water inflow f_n increases. Therefore, as long as no agreement has been reached, the most downstream agent will invoke his property rights on all the water resources and forbids his predecessors to consume any water.

In contrast to the ATS case, econometric estimation is only possible for the most downstream agent in applications where time-series data for x, f, e and values for costs and benefits are observed that conform to strict UTI. Then, similar as before, the estimation for agent n will be performed on (3.4). This suffices to obtain an estimate for the vector $d^{UTI} = (0, \ldots, 0, d_n^{UTI})$ of disagreement utilities.

3.3.2.2 Individual aspiration levels

Ambec and Sprumont (2002) define agent's i individual aspiration level as the maximal welfare that i would be able to achieve in the absence of all other agents, i.e., when agent i would be able to use the entire water inflow f_i^{UTI} at his own territory and the territories of all his upstream agents.[2] As noticed before, the individual aspiration levels are infeasible when the river contains at least two agents and the agents have to compromise on their aspiration levels in order to reach agreement. Despite infeasibility, individual aspiration levels often provide important reference points for individual decision makers. Also, Locke (1948) (see page 24) wrote: "Now, of those good things which nature hath provided in common, every one had a right, as hath been said, to as much as he could use, and property in all he could effect with his labor; all that his industry could extend to, to alter from the state nature had put it in, was his." In this tradition, each agent has a legitimate right to the individual aspiration level, but not to more.

Formally, when all agents invoke the UTI principle, the individual aspiration levels are obtained as follows. For every agent i, the inflow under disagreement f_i^{ASP} coincides with f_i^{UTI}, i.e., $f_i^{ASP} = e_i + \sum_{j \in P^i} e_j$, and the individual aspiration water use of agent i is given by

$$x_i^{ASP} = \arg\max_{x_i} b_i(x_i) - c_i(f_i^{ASP}, x_i), \text{ s.t. } x_i \le f_i^{ASP}, \tag{3.5}$$

where uniqueness is guaranteed by Assumption 3.1 and 3.2. This gives the aspiration-level utilities $d_i^{ASP} = b_i(x_i^{ASP}) - c_i(f_i^{ASP}, x_i^{ASP}) \ge 0$ for all agents $i \in N$. Notice that for all agents i located at a source of the river, i.e., $P^i = \emptyset$, we have $d_i^{UTI} = 0 < d_i^{ASP} = d_i^{ATS}$, for all 'middle' agents $i \in N \setminus \{n \cup i : P^i = \emptyset\}$ we have $d_i^{UTI} = 0 \le d_i^{ATS} < d_i^{ASP}$, and only for the most downstream agent n we have $d_n^{ATS} < d_n^{ASP} = d_n^{UTI}$. The following result extends the infeasibility of individual aspiration levels in Ambec and Sprumont (2002).

[2] Note that Ambec and Sprumont (2002) also define group aspiration levels for coalitions of agents, which does not appear in our analysis.

Proposition 3.3. *In the river model (N, R, u, e) it holds that $\sum_{i=1}^{n} d_i^{ASP} > w$.*

Although time-series data for inflows f^{ASP} can be trivially constructed from time-series data of e, the infeasibility of individual aspiration levels implies that no time-series data for x^{ASP} will ever be observed in reality. However, in case it is a priori known that reality conforms to ATS and time-series data for x, f, e and values for costs and benefits are observed, then econometric estimation as discussed in Section 3.3.1 is possible. From these estimations the aspiration-levels of water extraction x^{ASP} and the aspiration-level utilities $d^{ASP} = (d_1^{ASP}, \ldots, d_n^{ASP})$ can be computed.

3.4 The ANBS in the river sharing problem

Agents can improve on the inefficient disagreement outcomes associated with the principles of ATS, respectively strict UTI, by unanimity bargaining for joint river management. In this section, we apply the ANBS in order to mimic the outcome of such negotiations. This modeling choice can be justified by referring to the 1997 UN Convention that requires consent by all countries in the river basin. It is widely accepted that the ANBS captures unanimity bargaining.

We first establish the ANBS for a general river problem (N, R, u, e) with some maximum welfare w and an unspecified vector d of disagreement utilities with $\sum_{i\in N} d_i < w$. Next, we show that by the quasi-linear utilities of the agents the problem to find the ANBS can be decomposed into two smaller subproblems that facilitates its computation: first the utilitarian welfare maximum is computed yielding the efficient water use and second the monetary transfers according to the ANBS are computed. This gives a closed-form solution for the transfers. Finally, we discuss the political economy of property rights by analyzing the ANBS for disagreement utilities associated to the principles of ATS and strict UTI.

3.4.1 The bargaining solution

The origin of the asymmetric distribution of bargaining weights among all agents, as assumed by the ANBS, is outside the scope of the ANBS and is part of the axiomatization of the ANBS in e.g., Kalai (1977). In economic applications bargaining weights are often related to GDP, population sizes, political factors, military powers etc. This reflects that countries with, say, a larger GDP have much more at stake internationally as well as more financial means to maintain a large and well-trained corpse of diplomats and negotiators, or a large army to conduct geopolitics. The strategic bargaining literature underpins bargaining weights as either the probability of setting the agenda in random proposer bargaining or the differences in individual time-preferences, see e.g., Herrero (1989), Miyakawa (2006), Laruelle and Valenciano (2008) and Herings and Predtetchinski (2010). We regard the ANBS as a positive theory reflecting some underlying bargaining process.

Formally, the agents' bargaining weights are given by a vector $\alpha = (\alpha_1, ..., \alpha_n)$, where $\alpha_i \geq 0$ and $\sum_{i\in N} \alpha_i = 1$. In this section we further assume the disagreement utilities as exogenously given and impose that every agent i has a disagreement utility $d_i \geq 0$ and that $\sum_{i\in N} d_i < w$, where w is the maximum welfare that the agents can obtain in the river model (N, R, u, e). The nonnegativity condition is natural given that inaction gives zero utility. Given w and the vector d, the bargaining set consists of all utility vectors $u \in \mathbb{R}^n$ that are individually rational, thus $u_i \geq d_i$ for all i, and feasible, thus the sum of components is at most equal to w.

The ANBS seeks to maximize the asymmetric Nash product $\prod_{i=1}^n (u_i(f_i, x_i, t_i) - d_i)^{\alpha_i}$ under the constraints that the vector of water uses $x \in \mathbb{R}^n_+$, the vector of inflows $f \in \mathbb{R}^n_+$ and the vector of monetary transfers $t \in \mathbb{R}^n$ are feasible. This gives the following problem

$$\left.\begin{array}{ll} \max_{x,f\geq 0;t} & \prod_{i=1}^n \Big(b_i(x_i) - c_i(f_i, x_i) + t_i - d_i\Big)^{\alpha_i} \\ \text{s.t.} & f = e + (R - I)(e - x) \\ & Rx \leq Re, \ \ (p) \quad \text{and } \sum_{i=1}^n \ t_i \leq 0, \ \ (\lambda) \\ & b_i(x_i) - c_i(f_i, x_i) + t_i \geq d_i, \quad i = 1, \ldots, n \end{array}\right\}, \tag{3.6}$$

where $p \in \mathbb{R}^n$ and λ are the Lagrange multipliers for the water resource constraints and monetary transfers, respectively. Similar as for utilitarian welfare maximization, without additional assumptions on the benefit and cost functions, optimization program (3.6) may admit multiple maximizers. Again for ease of discussion and to avoid notational burden, we restrict ourselves to cases with a unique maximizer.[3] We have the following result.

Theorem 3.1. *Let x^*, $f^* = e + (R - I)(e - x^*)$ and t^* be the water allocation, the vector of inflows and the monetary transfers in the ANBS for the river sharing problem (N, R, u, e). Then x^* and f^* satisfy the first-order conditions*

$$G = R^\top P - (R - I)^\top F, \tag{3.7}$$

$$\text{with } G = \begin{bmatrix} b_1'(x_1) - \frac{\partial c_1(f_1,x_1)}{\partial x_1} \\ b_2'(x_2) - \frac{\partial c_2(f_2,x_2)}{\partial x_2} \\ \vdots \\ b_n'(x_n) - \frac{\partial c_n(f_n,x_n)}{\partial x_n} \end{bmatrix}, \ P = \begin{bmatrix} \frac{p_1}{\lambda} \\ \frac{p_2}{\lambda} \\ \vdots \\ \frac{p_n}{\lambda} \end{bmatrix} \text{ and } F = \begin{bmatrix} \frac{\partial c_1(f_1,x_1)}{\partial f_1} \\ \frac{\partial c_2(f_2,x_2)}{\partial f_2} \\ \vdots \\ \frac{\partial c_n(f_n,x_n)}{\partial f_n} \end{bmatrix},$$

and t^* is given by $t_i^* = \alpha_i \sum_{j=1}^n \left[b_j(x_j^*) - c_j(f_j^*, x_j^*) - d_j\right] - [b_i(x_i^*) - c_i(f_i^*, x_i^*) - d_i]$, $i = 1, ..., n$.

Note that the matrices R and $R - I$ defining the constraints in (3.6) reappear in (3.7), which relates the river geography directly to the ANBS. We can distinguish the effects of resource scarcity (P) from the effects of inflows on the cost of extraction (F).

Theorem 3.1 shows that the monetary transfer paid or received by agent i depends on his bargaining weight α_i of the aggregate net surplus $\sum_{j=1}^n \left(b_j(x_j^*) - c_j(f_j^*, x_j^*) - d_j\right)$ from cooperation minus his own improvement from cooperation $b_i(x_i^*) - c_i(f_i^*, x_i^*) - d_i$. Clearly his transfer is increasing in his bargaining weight, i.e., agent i pays less or receives more if he is assigned a larger bargaining weight since $\sum_{j=1}^n \left(b_j(x_j^*) - c_j(f_j^*, x_j^*) - d_j\right) > 0$.

Next we turn to Equation (3.7). For agent i, b_i' and $\frac{\partial c_i}{\partial x_i}$ are the marginal benefit of water use and the marginal cost of water extraction, respectively. Hence, $b_i' - \frac{\partial c_i}{\partial x_i}$ is his

[3]Here (3.6) specifies a convex set in the (x, f) space and the asymmetric Nash product function is strictly quasi-concave in utilities.

marginal net benefit of water extraction. Noticing that $R_{ji} = 1$ if and only if $j \in S^i \cup \{i\}$, it follows that the i-th row of System (3.7) can be written as

$$b'_i - \frac{\partial c_i}{\partial x_i} = \sum_{j=1}^{n} R_{ji}\frac{p_j}{\lambda} + \sum_{j=1, j\neq i}^{n} -R_{ji}\frac{\partial c_j}{\partial f_j} = \sum_{j\in S^i\cup\{i\}} \frac{p_j}{\lambda} + \sum_{j\in S^i} -\frac{\partial c_j}{\partial f_j}$$

$$= \sum_{j\in S^i\cup\{i\}} \frac{p_j}{\lambda} + \sum_{j\in S^i} \frac{\partial c_j}{\partial f_j}\frac{\partial f_j}{\partial x_i}, \tag{3.8}$$

where the last equality comes from the fact that $f_j = e_j + \sum_{i\in P^j}(e_i - x_i)$ and thus $\frac{\partial f_j}{\partial x_i} = -1$ for $j \in S^i$.

First notice from Equation (3.8) that the marginal net benefit of agent i in the optimum is independent of the bargaining weights α. Next notice that the first term of the right-hand side of (3.8) shows the impact that agent i imposes upon the physical availability of water for his successors. It reflects the resource scarcity of water for agent i and all his successors through the shadow prices p of the the local resource constraints $Rx \leq Re$, where $p_j > 0$ if $x_j \leq f_j$ is binding, $j \in N$. For agent i this term drops out if $x^*_j < f^*_j$ for all $j \in S^i \cup \{i\}$, i.e., neither the constraint of i nor any of his successors' constraint is binding. If $x^*_j = f^*_j$ for some $j \in S^i \cup \{i\}$, then p_j decreases when more local water resource e_j becomes available. When $p_k = 0$ for all $k = i$ and all agents k between i and j (and thus all the corresponding constraints are not binding), then the marginal net benefits of agent i, agent j and all agents between them will decrease because all these agents could consume some more water when more local water resource e_j becomes available. On the other hand, when $p_k > 0$ for $k = i$ or some agent k between i and j, then a higher local resource e_j does not allow agent i to consume more water and so the marginal net benefit of i does not change. More water inflow e_j and thus a lower p_j then induces a higher price p_k for agent $k = i$ or at least one other agent k between i and j. In this case more local inflow at j induces a higher shadow price, so relatively more scarcity, for at least one agent upstream of j.

The second term of the right-hand side of (3.8) is the sum of all externalities that agent i imposes upon the costs of extraction of all his successors. By assumption $\frac{\partial c_j}{\partial f_j} < 0$ and thus $\frac{\partial c_j}{\partial f_j}\frac{\partial f_j}{\partial x_i} > 0$ for every successor j of i. So, the negative externalities on the extraction costs of his successors lead to higher marginal net benefit for agent i. Hence, the consumption x^*_i in the optimum is lower than what i would like to consume when he is maximizing his own net benefit.

Since every individual term in the summation of the second right-hand term of Equation (3.8) is strictly positive and $S^j \subset S^i$ if $j \in S^i$, the next proposition holds, showing that the marginal net benefits are decreasing from upstream to downstream.

Proposition 3.4. *In the ANBS water allocation x^* of the river sharing problem (N, R, u, e), for every $i \in N$ and $j \in S^i$ it holds that $b'_i - \frac{\partial c_i}{\partial x_i} > b'_j - \frac{\partial c_j}{\partial x_j}$.*

The intuition is that, the closer agent i is located to one of the sources of the river, the more downstream successors experience such negative externalities from using an extra drop of water by agent i. Only the most downstream agent does not induce these externalities. Similarly, if agent i experiences water scarcity, i.e., $p_i > 0$, then all of his predecessors also experience water scarcity and this positive shadow price p_i will show up in their right-hand side of (3.8). This implies that the closer agent i is to the most-downstream location, he will have larger sets of predecessors and this agent's water scarcity is felt by more upstream agents.

Defining $b_i^s = b_i' - \frac{\partial c_i}{\partial x_i} - \sum_{j \in S^i} \frac{\partial c_j}{\partial f_j} \frac{\partial f_j}{\partial x_i}$ as the societal marginal net benefit of agent i, i.e., his own marginal net benefit minus the impact of x_i on the marginal extraction costs of his successors, we obtain by rearranging (3.8) that

$$b_i^s = \sum_{j \in S^i \cup \{i\}} \frac{p_j}{\lambda}, \qquad i \in N. \tag{3.9}$$

It follows immediately that the societal marginal net benefits are nonincreasing from upstream to downstream and they are all equal to each other if and only if $p_i = 0$ for all $i < n$.

Corollary 3.1. *In the ANBS water allocation x^* of the river sharing problem (N, R, u, e), for every $i \in N$ and $j \in S^i$ it holds that $b_i^s \geq b_j^s$ with at least one strict inequality if and only if $p_j > 0$ for some $j \neq n$.*

The above results generalize the results in Kilgour and Dinar (2001) and Ambec and Sprumont (2002) for the linear river sharing problem to general river geographies captured by R and externalities on the cost of extraction. They observe, as stated in Ambec and Sprumont (2002), that "the marginal benefits decrease (weakly) as one moves downstream and, if two agents have different marginal profits, some constraint must be binding between them." Corollary 3.1 shows the same result for the societal marginal net benefits, which include the marginal own extraction costs and the negative marginal externality costs of extraction on the successors of an agent. Finally, we remark that ANBS x^* and f^* cannot be implemented by a uniform water price.

3.4.2 Decomposition of the computation of the ANBS

In this section, we decompose the ANBS into two separate subproblems of which one has a closed-form solution. The first subproblem immediately arises from the following result. The water uses and inflows of the ANBS coincide with the utilitarian welfare maximizing water uses and inflows.

Theorem 3.2. *Let (x^*, f^*, t^*) be the ANBS for the river sharing problem (N, R, u, e). Then, $x^* = x^{UW}$, $f^* = f^{UW}$.*

The theorem implies that the aggregate net surplus $\sum_{i=1}^n [b_i(x_i^*) - c_i(f_i^*, x_i^*) - d_i]$ at the ANBS is equal to the aggregate net surplus $w - \sum_{i=1}^n d_i$ resulting from maximizing the utilitarian welfare. The intuition of this result is rather straightforward. One of the axioms of the ANBS requires Pareto efficiency and quasi-linear utility functions induce the utility possibility set $U = \{u \in \mathbb{R}^n | \sum_{i=1}^n u_i \leq w\}$. So, Pareto efficiency implies aggregate utilitarian welfare $\sum_{i=1}^n [b_i(x_i^*) - c_i(f_i^*, x_i^*)] \geq w$ and $(b_1(x_1^*) - c_1(f_1^*, x_1^*), \ldots, b_n(x_n^*) - c_n(f_n^*, x_n^*))$

$\in U$ implies the opposite weak inequality. This insight extends beyond the current application and will hold in general for the ANBS whenever individual utility functions are quasi-linear. For instance, it also holds in the static annual model with two seasons in Houba et al. (2013), who consider the externalities of the Mekong River as a green source of energy.

Theorem 3.2 can be related to the discussion on the Coase Theorem. The most well-known version states that, in the absence of transaction costs, Pareto efficiency arises independent of the assignment of property rights. Note that, in terms of axiomatic solutions, the Coase Theorem states a condition under which the efficiency axiom underlying the ANBS is justified and this axiom is always stated independently of the disagreement point. Hence, given how property rights are translated into disagreement points, the efficiency axiom underlying the ANBS is trivially independent of property rights. More interesting is that the Pareto efficient allocation of water at the ANBS is also independent of the disagreement point and, thus, independent of property rights. Given that we take uniqueness of x^{UW} and f^{UW} for granted, we also obtain the invariance version of the Coase Theorem: in the absence of transaction costs, the same physical allocation arises through negotiations independent of the assignment of property rights.

As a technical remark, we briefly address the case of multiplicity of maximizers to utilitarian welfare maximization (3.2) and ANBS (3.6). In that case, the result is that (x^{UW}, f^{UW}) is a maximizer of (3.2) if and only if there is a maximizer (x^*, f^*, t^*) of (3.6) such that $x^* = x^{UW}$ and $f^* = f^{UW}$. Furthermore, the invariance version of the Coase Theorem has that each maximizer is independent of property rights. We do not elaborate on these technicalities.

Since the maximization of utilitarian welfare already characterizes the Pareto efficient water uses x^* and inflows f^*, the next issue is to determine the transfers that maximize the Nash product given x^* and f^*. We stress once more that unanimity requires that each agent must obtain at least his disagreement utility, because otherwise agents who get less will deviate. Without proposing a formal procedure, within our simple framework this can be thought of as follows. As is common in international negotiations over treaties, the negotiations result has to be ratified afterwards by all the participants in the negotiations. If agent i's utility u_i from the treaty is lower than his disagreement utility d_i, this agent will not ratify and this will prevent the treaty from being implemented. Ratifying any treaty that will give an utility at least equal to the disagreement utility and rejecting otherwise, i.e., ratify if and only if $u_i \geq d_i$, is a Nash equilibrium strategy of this ratification process for every agent $i \in N$. This argument limits the set U of feasible utility vectors to the bargaining set $U^{IR}(d) = \{u \in U | u_i \geq d_i, i \in N\}$ of all feasible vectors satisfying individual rationality. Given x^* and f^*, the utility of agent i is given by $u_i(f_i^*, x_i^*, t_i) = b_i(x_i^*) - c_i(f_i^*, x_i^*) + t_i$, which we will write more conveniently as $u_i(f_i^*, x_i^*, t_i) = u_i(f_i^*, x_i^*, 0) + t_i$. Successively, we consider the following maximization problem with respect to the monetary transfers

$$\left.\begin{array}{ll} \max_{t \in \mathbb{R}^n} & \prod_{i=1}^n (u_i(f_i^*, x_i^*, 0) + t_i - d_i)^{\alpha_i}, \\ \text{s.t.} & \sum_{i \in N} t_i \leq 0, \quad \text{and} \quad u_i(f_i^*, x_i^*, 0) + t_i \geq d_i, \ i \in N. \end{array}\right\}. \tag{3.10}$$

We have the following result.

Theorem 3.3. *Let* $\hat{t}$ *be the solution of the maximization problem* (3.10)*. Then,* $(x^*, f^*, \hat{t})$ *coincides with the ANBS* (x^*, f^*, t^*) *for the river sharing problem* (N, R, u, e)*. Moreover,*

$$\hat{t}_i = d_i + \alpha_i \left(w - \sum\nolimits_{j=1}^{n} d_j \right) - u_i(f_i^*, x_i^*, 0),\ i = 1, ..., n.$$

From Theorem 3.2 and 3.3, it follows that the computation of the ANBS can be decomposed into two steps: in Step 1, we find the unique maximizer of (3.2) and, then, we may set $x^* = x^{UW}$ and $f^* = f^{UW}$. In Step 2, we determine $\hat{t}$, for which we have a closed-form solution given the Pareto efficient x^* and f^* of Step 1. Note that we rewrote the transfer when compared to Theorem 3.1. Agent i's utility in the ANBS is given by $d_i + \alpha_i(w - \sum_{j=1}^n d_j)$ and it is equal to the utility $u_i(f_i^*, x_i^*, 0)$ obtained from the use of water plus the monetary transfer $\hat{t}_i$. Since the welfare w is larger than $\sum_{j=1}^n d_j$, the monetary transfer of agent i is increasing in his bargaining weight α_i. In terms of our previous discussion of the Coase Theorem, the assignment of property rights does have welfare consequences for the agents through the disagreement point, because each agent's utility at the ANBS depends upon the disagreement point and the associated net surplus. Also the financial transfers depend upon the disagreement point.

Notice that for any agent i the financial transfer $\hat{t}_i$ obtained from solving the optimization problem (3.10) is increasing in d_i and decreasing in every d_j, $j \neq i$. A well-known result from bargaining theory states that an increase in agent i's disagreement utility improves this agent's bargaining position in the negotiations and, keeping the other disagreement utilities fixed, will result in an increase in his final utility. Since the ANBS satisfies the axiom of Pareto efficiency, agent i's improvement is at the expense of the other agents. Or, as our last result states, an increase in agent i's disagreement utility decreases each other agent j's final utility.

Although we treat the ANBS mainly as an approximation of the outcome of negotiations, it also has a normative interpretation in which the asymmetric Nash product is interpreted as a social welfare function, see e.g., Kaneko (1980). This interpretation requires a central agency with the authority to both impose and enforce policies, who evaluates river sharing policies according to such social welfare function, is able to assess all agents' true costs and benefit functions and assigns weights among the agents. The ANBS might be implemented by either quantity restrictions on water uses or regulation of water prices. Besides, the central agency should also have the authority to set the appropriate monetary transfers among agents through lump-sum taxes or subsidies. A potential institutional setup might be that the central agency decentralizes its policy to local governments. Then the local governments set local consumer prices and local producers claim their producer prices to local governments. All are according to the efficient allocation scheme set by the central agency. These institutional conditions are rather restrictive and seem to be more natural to guide water sharing among cities or states within a country. Nevertheless, proper application of the ANBS ensures a win-win policy because the central agency's optimal policy will be individual rational for all agents along the river, albeit that the gains may be asymmetrically distributed.

Finally, Chiappori et al. (2011) discuss and resolve several methodological issues in estimating and testing the Nash bargaining solution in economic applications. One key result is that imposing additional restrictions is necessary to recover the underlying structure of the Nash bargaining solution, in particular to estimate the utility functions and disagreement points. In our model, the assumption of quasi-linear utility functions provides such a restriction that allows for the decomposition discussed earlier and this decomposition enhances econometric estimation.

3.4.3 The political economy of property rights

In this section, we specify the disagreement utilities $d \in \mathbb{R}^n$ according to the different legal principles of ATS and strict UTI and we investigate and compare the resulting ANBS as obtained in the previous sections. These legal principles implicitly assign property rights among the agents. In any process in which agents decide on property rights before negotiating joint river management, each agent tries to invoke the legal principle that serves his best interest.

Recall from Section 3.3.2 that $d_i^{ATS} \geq 0 = d_i^{UTI}$ for $i = 1 \ldots, n-1$ and that $d_n^{ATS} < d_n^{UTI}$. For explanatory simplicity, we assume in this section that $d_i^{ATS} > 0$ for all $i < n$, so for all agents except agent n, the ATS disagreement utility is strictly higher than the strict UTI disagreement utility, only for agent n the opposite holds. However, rational agents are forward looking and are not interested in the disagreement utilities as such, but rather how these affect their final utility in the outcome of the negotiations.

By Proposition 3.1 and 3.2, the disagreement points d^{ATS} and d^{UTI} both belong to the utility possibility set U, but the bargaining sets $U^{IR}\left(d^{ATS}\right)$ and $U^{IR}\left(d^{UTI}\right)$ of individual rational utilities differ. According to Theorem 3.3, the final utilities under ATS, respectively strict UTI, become

$$u_i^{ATS} = d_i^{ATS} + \alpha_i \left(w - \sum\nolimits_{j=1}^{n} d_j^{ATS}\right), i = 1, \ldots, n, \quad \text{and} \tag{3.11}$$

$$u_i^{UTI} = \alpha_i \left(w - d_n^{UTI}\right), i = 1, \ldots, n-1, \quad u_n^{UTI} = d_n^{UTI} + \alpha_n \left(w - d_n^{UTI}\right). \tag{3.12}$$

From the formulas, we see two effects that can be related to the disagreement point d as the fall-back position in case the negotiations break down and the net surplus $w - \sum_{j=1}^{n} d_j$ that is bargained over. For all upstream agents $i = 1, \ldots, n-1$, a shift in legal principles from strict UTI to ATS increases agent i's fall-back position from $d_i^{UTI} = 0$ to d_i^{ATS} and, simultaneously, decreases agent n's fall-back position from d_i^{UTI} to d_i^{ATS}. So, in terms of fall-back positions, only agent n prefers the strict UTI principle. In terms of property rights, the intuition is that upstream agents get more rights over the water available, whereas the downstream agent looses his monopoly rights. However, the effect of a shift in legal principles from strict UTI to ATS has an ambiguous effect on the net surplus. It weakly increases the net surplus whenever $\sum_{j=1}^{n} d_j^{ATS} \leq d_n^{UTI}$, or $\sum_{j=1}^{n-1} d_j^{ATS} \leq d_n^{UTI} - d_n^{ATS}$. The latter inequality means that the aggregate gain in disagreement utility for agents $i = 1, \ldots, n-1$ is at most equal to the loss in disagreement utility $d_n^{UTI} - d_n^{ATS}$ for agent n. A larger net surplus means that the proportional gains from agreement are also larger and, evaluated in terms of the net surplus, all agents weakly prefer the net surplus of the ATS principle. Clearly, combining both effects under $\sum_{j=1}^{n} d_j^{ATS} \leq d_n^{UTI}$ immediately implies that all upstream agents $i = 1, \ldots, n-1$, prefer a shift in legal principles from strict UTI to ATS and only the most downstream agent n prefers the opposite shift. This can also be seen as the common intuition about who along the river prefers which of these two legal principles. The following result states the conditions under which this common intuition reverses for some upstream agents.

Theorem 3.4. *Agent n strictly prefers UTI to ATS. Agent i, $i < n$, strictly prefers strict UTI to ATS if and only if $d_n^{UTI} < \sum_{j=1}^n d_j^{ATS}$ and $\alpha_i > \frac{d_i^{ATS}}{\sum_{j=1}^n d_j^{ATS} - d_n^{UTI}}$.*

This result states that the most-downstream agent always prefers the strict UTI principle. The intuition is straightforward: this agent becomes the sole owner of all the water under strict UTI and if other agents want to use water they have to pay agent n. For the other agents, the answer depends upon their bargaining weights and the sizes of the net surplus under both legal principles. Since we already discussed common intuition, we concentrate on the case in which the net surplus under strict UTI is larger than the net surplus under ATS, i.e., $d_n^{UTI} < \sum_{j=1}^n d_j^{ATS}$. Then both effects of shifting legal principles, being the fall-back position and the net surplus, are opposite to each other and the total effect is ambiguous. If in addition agent i, $i < n$, is relatively strong in bargaining, which is reflected in a large bargaining weight, then this agent prefers the strict UTI principle as the principle defining initial property rights knowing that his bargaining power will ensure a share of a larger net surplus under strict UTI that compensates for his lower fall-back position under strict UTI.

Because agent n always obtains a larger utility under strict UTI when compared to ATS and the aggregate maximal welfare is w, there is always at least one agent $i\,(\neq n)$ who has to get a lower final utility under strict UTI than under ATS. For an international river that is shared by two countries, downstream prefers UTI and, consequently, upstream prefers ATS. For international rivers involving more countries, it is an empirical research question whether except country n, other countries are also better off under strict UTI than under ATS and if so, which countries.

3.5 The asymmetric Nash rationing solution

The individual aspiration levels when all agents invoke UTI, as defined in Section 3.3.2.2, lie above the Pareto frontier of the utility possibility set, hence these levels are infeasible and cannot be achieved. In this case, we treat the individual aspiration levels as a reference point in which a consensus among the agents requires each agent to bear some losses with respect to his aspiration level. The question then becomes on what compromise outcome the agents agree.

Many compromise solutions exist in the literature. In this section, we focus on Mariotti and Villar (2005), who study the problem of allocating utility losses among n agents, called the Nash rationing problem, which can be regarded as the translation of the Nash bargaining problem to a situation of compromising on utility losses. The Nash rationing solution is a symmetric set-valued solution and consists of the set of points that maximizes a weighted sum of utilities, in which weights are endogenously chosen so that all agents' weighted losses are equal. For problems with transferable utility, the solution is unique and coincides with the unique maximizer of the Nash rationing product. Translated into the river sharing problem, this product is defined as $\prod_{i=1}^n (d_i^{ASP} - u_i)$ and the Nash rationing solution is the unique maximizer of this product over the set of utility vectors

$u \in \mathbb{R}^n$ under the constraints $\sum_{j=1}^n u_j \geq w$ and $u_i \leq d_i^{ASP}$, $i = 1, \ldots, n$. Mariotti and Villar (2005) provide an axiomatization of the Nash rationing solution. The convergence results of equilibria in strategic bargaining models to the Nash bargaining solution in e.g. Binmore et al. (1986) indicates that a similar strategic underpinning of the Nash rationing solution is plausible. In particular, Houba (1997) analyzes fluctuating disagreement points from which an utopia point is constructed on which agents compromise in equilibrium.

In this section, we propose an asymmetric version of the Nash rationing solution. We do not provide an axiomatization, but given the similarities in the axiomatization results for the ANBS and the symmetric Nash rationing solution in Mariotti and Villar (2005), it is reasonable to conjecture that the asymmetric version with exogenous weights can also be axiomatized.[4] We postpone the interpretation of these weights. Formally, the weights are given by a vector $\rho = (\rho_1, ..., \rho_n)$, where $\rho_i \geq 0$ and $\sum_{i \in N} \rho_i = 1$. Distinguishing different notation for weights allows for flexibility.[5]

Given a weight vector ρ, we define the asymmetric Nash rationing solution (ANRS), as the solution of the maximization problem

$$\max_{(u_1,\ldots,u_n)} \prod_{i=1}^n (d_i^{ASP} - u_i)^{\rho_i}, \text{ s.t. } \sum_{j=1}^n u_j \geq w, \text{ and } u_i \leq d_i^{ASP}, \quad i \in N.$$

This convex program admits a unique maximizer, denoted u^{NRS}. Note that u_i^{NRS} is the utility level that each agent gets from the river sharing problem including the monetary transfer. Similar as before for the ANBS, agent i's monetary transfer closes the gap between his direct net benefit from water use $u_i(x_i^*, f_i^*, 0)$ and his final utility u_i^{NRS}. His transfer is given by $t_i^{NRS} = u_i^{NRS} - u_i(x_i^*, f_i^*, 0)$. Similar to Theorem 3.1, we obtain

$$u_i^{NRS} = d_i^{ASP} + \rho_i(w - \sum_{j=1}^n d_j^{ASP}), \quad i \in N. \tag{3.13}$$

Since according to Proposition 3.3, $w - \sum_{j=1}^n d_j^{ASP} < 0$, each agent gets a utility level below his individual aspiration level. Moreover, an agent's final utility is decreasing in the weight of the agent. Clearly, an interpretation of ρ in terms of bargaining weights makes no sense, because then a higher bargaining weight implies the counter-intuitive result that this agents gets a lower utility. Instead, the weight of an agent represents the responsibility of this agent, namely the more weight we put on agent i, the more responsibility this agent has to take in making sacrifices to reach a compromise. More responsibility results in a lower utility for an agent. Because the utility is decreasing in the weight, it might even happen that the utility in the ANRS falls below zero that an agent can guarantee himself by blocking agreement and inaction of refraining from using water. The following example illustrates this.

Example 3.2. Consider the case that the tributaries originating at location 1 and 2 merge before location 3. The agents' responsibility weights are given by $\rho = (\frac{3}{5}, \frac{1}{5}, \frac{1}{5})$.

[4]Personal communication with professor Mariotti confirmed this conjecture.

[5]For example, responsibility weight ρ_i may be inversely related to bargaining weight α_i.

The benefit functions, cost functions and water resources are given by,

$$b_i(x_i) = \sqrt{x_i},\ c_i(x_i) = x_i^2,\ \text{and } e_i = 1,\ \text{for } i = 1, 2,$$
$$b_3(x_3) = 16\sqrt{x_3},\ c_3(f_3, x_3) = x_3^2/f_3 \text{ and } e_3 = 0.$$

The maximum utilitarian welfare is $w = 20.7341$ and by application of (3.5), we obtain $d^{ASP} = (0.4725, 0.4725, 20.6274)^\top$. Then, formula (3.13) applied to agent 1 implies $u_1^{NRS} = 0.4725 + 0.6(20.7341 - 20.6274 - 0.4725 - 0.4725) = -0.0305 < 0$. □

Given that the utility of inaction is 0, the question is whether an agent who has to compromise on a negative utility according to (3.13) is willing to accept the agreement. Without his consent, the agreement fails unanimity. In terms of the ratification process of international treaties mentioned in Section 3.4.2, ratifying any treaty that will give an utility of at least equal to zero and rejecting otherwise, is a Nash equilibrium strategy of the ratification process for every agent $i \in N$. Therefore, it is natural and, as our example makes clear, necessary to impose the participation constraint $u_i \geq 0$ for every $i \in N$ in the maximization problem to find the Nash rationing solution. Adding the nonnegativity constraints to the Nash rationing solution complicates the maximization problem, however similar results obtain, except that now we have a boundary solution. For the maximization problem including the participation constraints $u_i \geq 0$, $i \in N$, let $T \subset N$ be the set of agents $j \in N$ that receive a utility $u_j^{NRS} > 0$ at the ANBS. Then, without going into details, we obtain for the Nash rationing solution with nonnegativity constraints that $u_i^{NRS} = 0$ if $i \in N \setminus T$, and

$$u_j^{NRS} = d_j^{ASP} + \frac{\rho_j}{\sum_{k \in T} \rho_k} \left(w - \sum\nolimits_{k \in T} d_k^{ASP} \right),\ \text{if } j \in T. \tag{3.14}$$

So, the agents in T split the deficit with respect to the total aspiration utilities of the agents in T according to their relative weights within this group. For $T = N$, Equation (3.14) coincides with (3.13).

The econometric estimation of asymmetric Nash rationing can be performed similarly as discussed in Section 3.4.2, but a modification is needed when binding participation constraints are present: after the cost and benefit functions are estimated and time series for the aspiration level utilities have been constructed, agents corresponding to approximately binding participation constraints are excluded to obtain the set T. Then, proceeding as in Section 3.4.2, all weights ρ_j, $j \in T$, can be estimated from (3.14). For all other weights ρ_i, $i \in N/T$, there is some freedom in estimating these weights due to the fact that the right-hand side of (3.13) is non-positive under the normalization $\sum_{i \in N} \rho_i = 1$.

3.6 Two numerical examples

In this section, we provide two numerical examples to illustrate the ANBS under different legal principles regarding the disagreement point in the International Water Law.

3.6.1 Example 1: two agents

Suppose that only two agents are positioned along the river with $e_1 = 1$, $e_2 = 0$ and the benefit functions and cost functions given by

$$\begin{aligned} b_1(x_1) &= \tfrac{1}{4}\sqrt{x_1},\ c_1(x_1) = \tfrac{1}{16}x_1^2, \text{ and} \\ b_2(x_2) &= \tfrac{1}{2}\sqrt{x_2},\ c_2(f_2, x_2) = \tfrac{x_2^2}{f_2}, \text{ where } f_2 = 1 - x_1. \end{aligned}$$

By application of Theorem 3.2, the maximal utilitarian welfare $w = 0.3102$ is attained at $x_1^* = 0.55$ and $x_2^* = 0.15$ with associated utilities $u_1^* = 0.1665$ and $u_2^* = 0.1437$. Note that this step does not involve monetary transfers.

Application of Equations (3.11) and (3.12) requires providing the disagreement points first. Under the ATS principle, application of (3.3) yields the disagreement point $d^{ATS} = (0.1875, 0)^\top$. Similarly, under the UTI principle of Section 3.3.2.1, application of (3.4) yields water uses $x_1^{UTI} = 0$ and $x_2^{UTI} = 0.25 < 1 = f_i^{UTI}$ from which we obtain the disagreement point $d^{UTI} = (0, 0.1875)^\top$. Figure 3.1 illustrates that different disagreement points give different bargaining sets, where the utility for agent 1 (2) is positioned on the horizontal (vertical) axis. In this figure, region A is the bargaining set under the ATS principle. For any pair of bargaining weights, the vector of ANBS utility levels according to (3.11) end up on the segment cd. Region B is the bargaining set under the UTI principle of Section 3.3.2.1 and the vector of ANBS utility levels specified by (3.12) end up on the segment ab. Independent of the bargaining weights, upstream agent 1 always prefers the ATS principle and downstream agent 2 the strict UTI principle.

In terms of Ambec and Sprumont (2002), the downstream incremental solution satisfies the ATS principle and no group of agents can achieve more than its group surplus. For $n = 2$, their solution maximizes agent 2's utility while keeping agent 1 at his disagreement utility under ATS, i.e., $u_1 = d_1^{ATS} = 0.1875$ and $u_2 = w - d_1^{ATS} = 0.1227 > d_2^{ATS}$. This is point c in Figure 3.1.

If both agents invoke the UTI principle of Section 3.3.2.2, application of (3.5) implies the unattainable aspiration level $d^{ASP} = (0.1875, 0.1875)^\top$, as Figure 3.1 illustrates. Given any pair of responsibility weights, each agent's utility level in the ANRS follows from Equation (3.13). In order to reach agreement, each agent has to bear utility losses to end up on the segment bc.

In the above example, when considering the Nash rationing solution of Mariotti and Villar (2005), we end up with the middle point of the segment bc which minimizes the weighted sum of individual losses and weights are chosen so that all individual weighted losses are equal. In this situation, we have equal weights for both agents, i.e., $(\frac{1}{2}, \frac{1}{2})$, since one unit increase of the utility level for agent 1 must decrease the utility level of agent 2 by 1 unit as well.

3.6.2 Example 2: three agents

We continue with Example 3.2 only that we change the benefit function of agent 3 into $b_3(x_3) = \sqrt{x_3}$, maintain $c_3(f_3, x_3) = x_3^2/f_3$ and that we do not specify the weights. Then, the maximum utilitarian welfare is given by $w = 1.4539$. Under ATS, application of (3.3) yields the disagreement point $d^{ATS} = (0.4725, 0.4725, 0.5021)^\top$. Under strict UTI, the

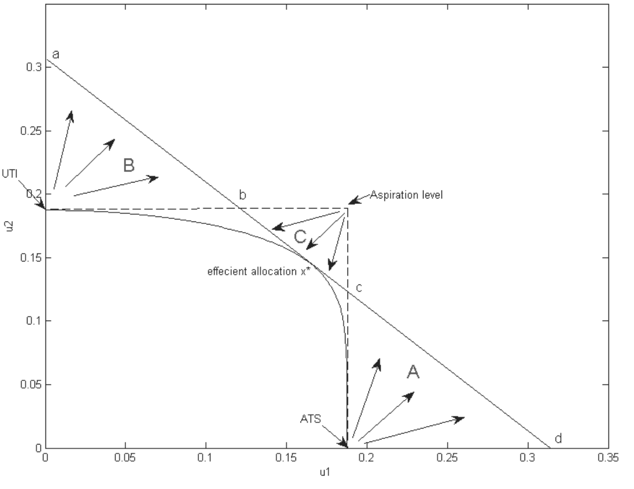

Figure 3.1: The asymmetric Nash solutions for the ATS, strict UTI principle and the individual aspiration levels in Example 3.6.1 (two agents).

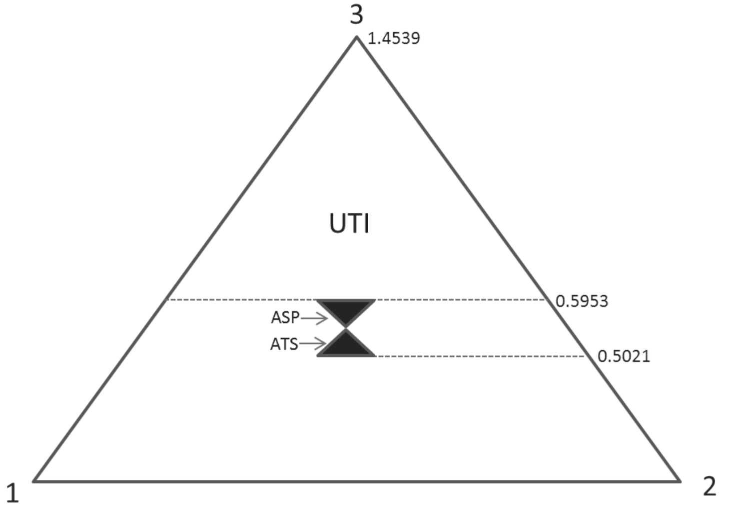

Figure 3.2: The asymmetric Nash solutions for the ATS, strict UTI principle and the individual aspiration levels in Example 3.6.2 (three agents).

Table 3.1: Utilities of the agents under ATS, UTI and ASP

Legal Principles	Surpluses or Losses	Utility for Agent i, $i = 1, 2$	Utility for Agent 3
UTI	0.8586	0+α_i 0.8586	0.5953+α_3 0.8586
ATS	0.0068	0.4725+α_i 0.0068	0.5021+α_3 0.0068
ASP	0.0864	0.4725-ρ_i0.0864	0.5953-ρ_3 0.0864

disagreement point is given by $d^{UTI} = (0, 0, 0.5953)^\top$. The vector of individual aspiration levels is given by $d^{ASP} = (0.4725, 0.4725, 0.5953)^\top$. Note that in this situation, we have

$$d_i^{ASP} = d_i^{ATS}, \text{ for } i = 1, 2;\ d_3^{ASP} = d_3^{UTI}.$$

In Figure 3.2, we draw the set of possible utility allocations for three agents in the simplex. The small upward-pointing triangular is the bargaining set under ATS, and under individual aspiration levels the bargaining set is the downward-pointing triangular. The large upper triangle, in which agent 3 gets at least a utility of 0.5953, is the utility bargaining set under strict UTI.

Given the weights, the Nash solution utilities of the agents under the different legal principles are given in Table 3.1. From this table, we see that agent 3 always prefers strict UTI to ATS, which confirms Theorem 3.4. This can also be deduced from Figure 3.2, where agent 3's utility in any utility vector in the large upper triangular is larger than this agent's utility in any utility vector in the small upward-pointing triangular. From Table 3.1, we see that agent 3 prefers strict UTI to ASP. To see this, first recall that $d_3^{ASP} = d_3^{UTI}$. Then, agent 3 prefers any share of the positive net surplus under strict UTI on top of his disagreement utility under this principle to any compromise under ASP that gives him less than his aspiration level.

Table 3.1 also implies that agent $i = 1, 2$ prefers strict UTI to ATS if his bargaining weight $\alpha_i > 0.5547$, where the lower bound is equal to the threshold stated in Theorem 3.4. Then, agent i can compensate the lower disagreement utility $d_i^{UTI} = 0$ (when compared to the more favorable $d_i^{ATS} = 0.4725$) with his share from the larger net surplus $w - d_3^{UTI} = 0.858\,6$ (compared to $w - \sum_{i=1,2,3} d_i^{ATS} = 0.0068$). Table 1 also shows that agents 1 and 2 prefer negotiations under the ATS principle to compromising under the ASP, which is due to $d_i^{ASP} = d_i^{ATS}$ for $i = 1, 2$.

3.7 Conclusion

In this chapter, we investigate unanimity bargaining among multiple agents for sharing a river and how several principles from International Water Law affect the bargaining outcome. We allow for a general river geography and general cost functions that depend upon river inflow and own extraction. To capture the bargaining, we apply asymmetric versions of the Nash bargaining and Nash rationing solution, which each yields an efficient agreement. One key finding is that the efficient water allocations are completely determined by the water resources, the river geography and the maximal utilitarian welfare. Under ATS and strict UTI, the disagreement outcome is feasible and inefficient.

Given the efficiency sets the unique water allocation, the remaining issue boils down to the monetary transfer. The negotiated joint management and financial compensations form a win-win outcome when compared to disagreement. Also, we derive conditions under which the common intuition reverses that all upstream and midstream countries always prefer the ATS principle to the strict UTI principle. This can only occur when some of these countries have sufficient bargaining power. Under the individual aspiration or utopia levels, these levels are no longer feasible and all agents have to compromise on their utopia levels in order to reach agreement. In this situation, the weights fail an interpretation as bargaining weights and should be interpreted as weights of responsibility in compromising. Higher responsibility weights require larger sacrifices. In terms of utopia levels, agreements imply incurring losses and the win-win feature is lost.

The analysis in this chapter can be generalized in several directions. Firstly, the ANBS framework is rich enough for further investigation of other principles from International Water Law to investigate how these affect the river sharing problem. For instance, another important principle is the one of fair and equitable optimal water use, as analyzed in Ambec and Ehlers (2008). In this reference, it is shown how water can be allocated equitable as assigned property rights before the agents trade water against money in a Walrasian competitive market. Then, voluntary trade moves the equitable water allocation towards the efficient water allocation. Secondly, alternative bargaining solutions from the literature may be considered. For example, the Kalai-Smorodinsky solution under ATS or strict UTI is already implicitly analyzed in our analysis, because for transferable utility this solution coincides with the symmetric Nash bargaining solution. Thirdly, given that every agent may invoke the legal principle that serves him best, an international political process to arrive at a compromise solution over different legal principles in order to establish legal principles may be needed first before to reach agreement on water and financial compensations. Fourthly, the implementation of the efficient water allocation needs further elaboration and we already discussed some rather strong institutional conditions under which it can be implemented. An alternative route for implementation of an equitable (final) allocation through mechanism design in a common pool is suggested in Ambec and Ehlers (2008) and we leave extension of this procedure to river sharing problems for future research. Finally, given empirical data, a more ambitious goal is to statistically estimate the bargaining (responsibility) weight in the ANBS (ANRS) from international river treaties, or at the national level, water allocation between provinces. Although river data is often difficult to get, some countries do publish suitable data. We discussed how our results may enhance econometric estimation in river sharing problems.

3.8 Appendix with proofs of Chapter 3

Proof of Proposition 3.1

The recursive ATS solution (x_i^{ATS}, f_i^{ATS}) satisfies all water resource constraints $x^{ATS} \leq f^{ATS}$ and $f^{ATS} = e + (R - I)\left(e - x^{ATS}\right)$. Therefore, it is a feasible solution of

$$\max_{x,f\geq 0} \sum_{i=1}^{n} (b_i(x_i) - c_i(f_i, x_i)) \text{ s.t. } Rx \leq Re, \; f = e + (R-I)(e-x).$$

Hence, $\sum_{i=1}^{n} d_i^{ATS} \leq w$. By Assumption 3.2, the recursively derived local optima (x_i^{ATS}, f_i^{ATS}) fail to be the maximizer of (3.2), because these do not internalize the externalities on the costs of extraction, i.e., $\frac{\partial c_i(f_i,x_i)}{\partial f_i} < 0$. Hence, $\sum_{i=1}^{n} d_i^{ATS} < w$. QED.

Proof of Proposition 3.3

By definition of (3.2), we have

$$\begin{aligned} w &= \sum_{i=1}^{n}(b_i(x_i^{UW}) - c_i(f_i^{UW}, x_i^{UW})) < \sum_{i=1}^{n}(b_i(x_i^{UW}) - c_i(f_i^{ASP}, x_i^{UW})) \\ &\leq \sum_{i=1}^{n}(b_i(x_i^{ASP}) - c_i(f_i^{ASP}, x_i^{ASP})) = \sum_{i=1}^{n} d_i^{ASP}. \end{aligned}$$

The strict inequality comes from the fact that $c_i(f_i, x_i)$ is decreasing in f_i and $f_i^{UW} < f_i^{ASP}$. QED.

Proof of Proposition 3.4

Without loss of generality, renumber the agents such that agent $i+1$ is agent i's downstream neighbor. By the tree structure of the river, $i+1 \in S^i = \{i+1\} \cup S^{i+1}$ and combined with (3.8) we obtain

$$\begin{aligned} b_i' - \frac{\partial c_i}{\partial x_i} &= \sum_{j\in S^i\cup\{i\}} \frac{p_j}{\lambda} - \sum_{j\in S^i} \frac{\partial c_j}{\partial f_j} = \underbrace{\frac{p_i}{\lambda} - \frac{\partial c_{i+1}}{\partial f_{i+1}}}_{>0} + \sum_{j\in S^{i+1}\cup\{i+1\}} \frac{p_j}{\lambda} - \sum_{j\in S^{i+1}} \frac{\partial c_j}{\partial f_j} \\ &> \sum_{j\in S^{i+1}\cup\{i+1\}} \frac{p_j}{\lambda} - \sum_{j\in S^{i+1}} \frac{\partial c_j}{\partial f_j} \overset{(3.8)}{=} b_{i+1}' - \frac{\partial c_{i+1}}{\partial x_{i+1}}. \end{aligned}$$

Recursive repetition of these arguments implies the stated result. QED.

Proof of Theorem 3.1

After substitution of f, we define $M = \Pi_{i=1}^{n}(b_i(x_i) - c_i(e_i + \sum_{j\in P^i}(e_j - x_j), x_i) + t_i - d_i)^{\alpha_i}$ for notational convenience. Because the asymmetric Nash product is equivalent to a Cobb-Douglas utility function, all individual rationality constraints $b_i(x_i) - c_i(f_i, x_i) + t_i \geq d_i$

will hold with a >-sign in the maximum. Without loss of generality, let $L(x,t,p,\lambda)$ denote the Lagrangian with shadow prices p and λ as defined (3.6). Then, the first-order conditions for x^* and t^* read

$$\frac{\partial L}{\partial x_i} : \alpha_i \frac{M}{b_i - c_i + t_i - d_i}(b_i' - \frac{\partial c_i}{\partial x_i}) + \sum_{j \in S^i} \alpha_j \frac{M}{b_j - c_j + t_j - d_j} R_{ji} \frac{\partial c_j}{\partial f_j} - \sum_{j \in S^i \cup \{i\}} R_{ji} p_j = 0,$$
$$\frac{\partial L}{\partial t_i} : \alpha_i \frac{M}{b_i - c_i + t_i - d_i} - \lambda = 0.$$

Writing the first n equations into matrix form, we obtain

$$R^T P = G + (R^T - I)F,$$

where P, G and F are stated in Theorem 3.1. With respect to the monetary transfers, we have

$$\frac{M}{b_i - c_i + t_i - d_i}\alpha_i = \lambda, \; i = 1, \ldots, n.$$

Dividing the equation for $i = 1$ by the one for i, we obtain

$$t_i = \frac{\alpha_i}{\alpha_1}(b_1 - c_1 - d_1) + \frac{\alpha_i}{\alpha_1} t_1 - (b_i - c_i - d_i).$$

This establishes a relationship between t_i and t_1 for all $i \geq 2$. Substitution of these expressions in $\sum_{i=1}^n t_i = 0$ yields,

$$t_1 + [\frac{\alpha_2}{\alpha_1}(b_1 - c_1 - d_1) + \frac{\alpha_2}{\alpha_1} t_1 - (b_2 - c_2 - d_2)] + ... + [\frac{\alpha_n}{\alpha_1}(b_1 - c_1 - d_1) + \frac{\alpha_n}{\alpha_1} t_1 - (b_n - c_n - d_n)] = 0.$$

From which, t_1 can be solved as,

$$t_1 = \alpha_1 \sum_{j=1}^n (b_j - c_j - d_j) - (b_1 - c_1 - d_1).$$

Similarly, we obtain

$$t_i = \alpha_i \sum_{j=1}^n (b_j - c_j - d_j) - (b_i - c_i - d_i).$$

QED.

Proof of Theorem 3.2

It suffices to show that the first-order conditions coincide with those for the ANBS given in the proof of Theorem 3.1. After substitution of f, we have to solve

$$\max_x \sum_{i \in N} \left(b_i(x_i) - c_i(e_i + \sum\nolimits_{j \in P^i} (e_j - x_j), x_i) \right), \text{ s.t. } Rx \leq Re \;\; (\tilde{p}).$$

where $\tilde{p}$ is the vector of Lagrange multipliers. The Lagrangian is given by

$$\tilde{L}\left(x,\tilde{p}\right)=\sum_{i\in N}\Big(b_i(x_i)-c_i(e_i+\sum\nolimits_{j\in P^i}(e_j-x_j),x_i)\Big)+\tilde{p}^{\top}(Re-Rx),$$

Taking first-order conditions, for x_i, we have

$$b_i^{'}-\frac{\partial c_i}{\partial x_i}+\sum_{j=1,j\neq i}^{n}R_{ji}\frac{\partial c_i}{\partial f_i}-\sum_{j=1}^{n}R_{ji}\tilde{p}_j=0.$$

We compare these with the first-order conditions for the ANBS

$$\frac{\partial L}{\partial x_i}:\alpha_i\frac{M}{b_i-c_i+t_i-d_i}(b_i^{'}-\frac{\partial c_i}{\partial x_i})+\sum_{j=1,j\neq i}^{n}(\alpha_j\frac{M}{b_j-c_j+t_j-d_j})R_{ji}\frac{\partial c_j}{\partial f_j}-\sum_{j=1}^{n}R_{ji}p_j=0.$$

Let $\alpha_i\frac{M}{b_i-c_i+t_i-d_i}=\lambda$. Then, a simple normalization of the Lagrange multipliers ($\frac{p_j}{\lambda}$) will get the stated result. QED.

Proof of Theorem 3.3

The Lagrange function for the maximization problem is[6]

$$L=\prod_{i=1}^{n}(u_i(f_i^*,x_i^*,0)+t_i-d_i)^{\alpha_i}-\lambda\sum_i t_i.$$

The first-order conditions read

$$\frac{\partial L}{\partial t_i}:\alpha_i\frac{M}{u_i(f_i^*,x_i^*,0)+t_i-d_i}-\lambda=0.$$

For $j\neq 1$, we have

$$t_j=\frac{\alpha_j}{\alpha_1}\left[u_1(f_1^*,x_1^*,0)-d_1\right]+\frac{\alpha_j}{\alpha_1}t_1-\left[u_j(f_i^*,x_i^*,0)-d_j\right].$$

This establishes a relationship between t_j and t_1 for all $j\geq 2$. Substitution of these expression into $\sum_{i=1}^{n}t_i=0$ and solve for t_1, we obtain,

$$\hat{t}_1=\alpha_1\sum_{j=1}^{n}\left[u_j(f_j^*,x_j^*,0)-d_j\right]-\left[u_1(f_1^*,x_1^*,0)-d_1\right].$$

It remains to check for the individual rationality constraint. Indeed,

$$u_1(f_1^*,x_1^*,0)+\hat{t}_1=d_1+\alpha_1(w-\sum_{j=1}^{n}d_j)>d_1$$

since $w>\sum_{j=1}^{n}d_j$. Similar results follow for $\hat{t}_j$ for all $j\geq 2$. Hence, $\hat{t}$ coincides with t^* stated in Theorem 3.1. QED.

[6]We omit the individual rationality constraint in the Lagrange function. Later on we will check for this.

Proof of Theorem 3.4

For agent n, we have

$$u_n^{UTI} > u_n^{ATS} \Longleftrightarrow (1-\alpha_n)\left(d_n^{UTI} - d_n^{ATS}\right) > -\alpha_n \sum_{j=1}^{n-1} d_j^{ATS}.$$

The inequality always holds, because the left-hand side is positive and the right-hand side is at most 0. For agent i, we have

$$u_i^{UTI} > u_i^{ATS} \Longleftrightarrow \alpha_i \left(\sum_{j=1}^{n} d_j^{ATS} - d_n^{UTI}\right) > d_i^{ATS}.$$

There are two cases to consider. First, for $\sum_{j=1}^{n} d_j^{ATS} \leq d_n^{UTI}$, the left-hand side becomes non-positive and we obtain $d_i^{ATS} < 0$, which violates that d_i^{ATS} is individual rational with respect to inaction $u_i(f_i, 0, 0) = 0$. Second, for $\sum_{j=1}^{n} d_j^{ATS} > d_n^{UTI}$, we obtain

$$u_i^{UTI} > u_i^{ATS} \Longleftrightarrow \alpha_i > \frac{d_i^{ATS}}{\sum_{j=1}^{n} d_j^{ATS} - d_n^{UTI}}.$$

Combining both cases implies that the lower bound on α_i and the condition for the second case are both necessary and sufficient for $u_i^{UTI} > u_i^{ATS}$. QED.